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PERIOD CHANGES OF THE SATELLITE 1960 EPSILON 3 IN 1963/64 AS DEDUCED FROM OBSERVATIONS WITHIN THE INTEROBS PROGRAM

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PERIOD CHANGES OF THE SATELLITE 1960ε3 IN 1963/64 AS DEDUCED FROM OBSERVATIONS WITHIN THE INTEROBS PROGRAM*

by

E. ILLÉS-ALMÁR and I. ALMÁR

Zusammenfassung. Aus [7] wurden die Beobachtungen des Satelliten 1960 ε3 (Kabine Sputnik 4) ausgewählt und die besten 39 Durchgänge, die sich zwischen 21 August 1963 und 27 Sept. 1964 verteilen, bearbeitet. Zuerst berechnete eine elektronische Rechenmaschine nach der Methode [9] die zu 393 simultanen Vermessungen gehörigen *XYZR* Raumkoordinaten des Satelliten. Dann wurde (nach einem Vorschlag von I. D. Shongolowitsch) bei jedem Durchgang die Durchgangszeit des Satelliten über einen ausgewählten geographischen Breitenkreis bestimmt (Tab. 3). Insofern die Periode sich gleichmässig änderte, konnte die Beschleunigung pro Umlauf $\frac{dP}{dn}$ mittels der Abweichungen zwischen den beobachteten und den mit einer konstanten Periode berechneten Durchgangszeiten bestimmt werden. Es wurden für 31 Durchgänge (267 Raumpunkte enthaltend) Periodenänderungen $\frac{dP}{dn}$ berechnet (Tab. 4). In allen beobachteten Zeitabständen war die Periodenänderung konstant, mit Ausnahme der ersten, wo am 26 August 1963 der $\left|\frac{dP}{dn}\right|$ Wert — parallel mit einer Verminderung der Sonnenfleckenzahl und der Gesamtfläche der Sonnenflecken — plötzlich abgenommen hat (Tab. 5, Abb. 2—8). Diese Änderung war sechsmal grösser als der Standardfehler der vorherigen $\frac{dP}{dn}$ Werte und entsprach einer Verminderung der Luftdichte in der Perigeumhöhe (255 km) um 31%. Es ist bemerkenswert, dass die $\left|\frac{dP}{dn}\right|$ Werte in Juli und Sept. wesentlich kleiner waren als die früheren. Diese Verminderung konnte auch unabhängig davon durch den zeitlichen Verlauf der Umlaufszeit bewiesen werden. Dieser Umstand hängt wahrscheinlich auch mit der Abnahme der Sonnenaktivität zusammen und daraus folgt der prolongierte Untergang des Satelliten.

INTRODUCTION

Air drag is playing a vital role among perturbing forces acting upon the motion of artificial satellites. Its effect becomes visible in the decrease of a, e and the period of revolution (P), these changes being connected with air density in the region near perigee. The decrease of the nodal period of revolution, easily measurable by observing the satellite's reappearance above a given geographic latitude, yealds a suitable tool for the investigation of the influence of air drag on satellite motion. As the deviations between observed and calculated transit times (O—C), on the one hand, and the rate of period

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change, on the other hand, are closely connected, even a regular tracking network of comparatively small extent might produce valuable data for atmospheric physics. Experience shows, however, that the density of upper atmosphere depends on the intensity of solar activity, hence satellite orbit studies may lead to a better understanding of geophysical influences of the Sun in general.

Several successful attempts have been made to utilize optical observations — including also rough visual fixes available in large quantities — in order to investigate changes in the orbital elements of artificial satellites [1]. As optical observations in the general case give only the direction of the satellite with respect to the tracking station, the determination of orbital elements is usually carried out by means of classical celestial mechanics based on a set of observations and on the method of successive approximation (Laplace, Gauss). From an adequate number of optical fixes orbital elements can be obtained, referring to the whole time interval during which the observational material has been collected. The method is more effective if the material is of higher accuracy, more extensive, and covers a longer portion of the whole orbit [2]. In practice, however, because of the quick changes in the elements of artificial satellites due to deviations of the Earth's gravitational field from a central field and also due to air drag, some assumptions must be introduced concerning the direction and speed of changes in the orbital elements in the period under investigation.

If perfect synchronism of observations carried out from a base line of appropriate length on the same satellite can be guaranteed in some way or another, the instantaneous position in space of the satellite is fixed. In this case orbit studies which are based on a limited number of observations are possible. This possibility means, in a fortunate case, a shorter time interval during which the shift of orbital elements may be either easily taken into account or neglected. This fact, important mainly for low-orbit satellites, gives an opportunity of studying even quick changes of certain orbital elements (e.g. period of revolution).

The synchronism of optical fixes may be assured, in general, by several methods:

a). If short light signals are emitted by a satellite at definite times (e.g. ANNA—1B), the position of the satellite with respect to the stars can be photographed from stations at both ends of a base line. This is the best solution of the problem, but owing to technical difficulties it cannot be used frequently.

b) If the camera control systems are connected so that they are activated by the same time signal, then every satellite illuminated by the sun can be photographed simultaneously from the tracking stations. The drawback of the method is that this perfect synchronism encounters serious technical difficulties.

c) If the apparent path of the satellite is supposed to be a great circle, pairs of simultaneous observations may be selected quite independently of time statements [3]. The topocentric radius vector M_2S belonging to a given topocentric radius vector M_1S must lie, however, in the plane defined by the known radius vector M_1S and the base line M_1M_2, and also in the plane described by the apparent path of the satellite (notations see on Fig. 1). The line of intersection of these planes fixes the direction of the satellite from

M_2 at a specified moment. One drawback of this method is that it requires extensive calculations.

d) An interpolation can be used to select a pair of simultaneous observations, frequent and consecutive from at least one of the stations and sporadic from the other, occurring within the same time interval. It is well known that the apparent orbit of an artificial satellite is quite smooth within an interval of less than a minute. Therefore this method has the advantage of reducing the random errors in the observations, since the interpolated values represent mean values of a set of observed azimuthal or equatorial coordinates, provided the orbit is smooth. The method was used in both the Soviet and American triangulation programs [4], [5]; it has been introduced by M. Ill to visual observations [6].

In order to increase the number of simultaneous observations a special network of Hungarian, German, later Soviet, Bulgarian, Polish and Roumanian tracking stations was established in 1961/62 under the designation INTEROBS. It has been organized, according to its coordinator Mr. M. Ill, for the purpose of performing serial observations within previously fixed time intervals on certain artificial satellites. All observations obtained during these intervals („campaigns") were summarized and published at Baja (Hungary) [7]. Several methods have been suggested [8] to evaluate simultaneous observations selected from this material by means of graphic interpolation. The aim of the present paper is to investigate tracking data on the cabin of Sputnik 4 (1960 ε3) using a combination of methods proposed at the conference of satellite observers at Riga in 1965.

Our experiences have shown that the observational material of the first INTEROBS cycles, collected mainly in Eastern Europe, did not enclose a portion of the orbit long enough to allow exact determination of all orbital elements (e and ω in particular) separately for every revolution. Hence, following a proposal of I. D. Zhongolovich, we intended only to study the changes of the nodal period of revolution of the satellite.

From the material available simultaneous observations carried out in the following intervals have been selected: 21—30 Aug. 1963, 17 Feb., 10—13 Apr., 8—13 June, 7—12 July and 18—27 Sept. 1964.

Evaluation has been limited to the following transits:

TABLE 1

Date	No. of transit in [7]	Number of sim. pairs of observations
1963		
21 Aug.	2	6
22	4	15
23	5	9
23	6	18
24	7	1
24	8	8
25	9	2
26	12	9
27	13	8
27	14	4

TABLE 1 (cont.)

Date	No. of transit in [7]	Number of sim. pairs of observations
29	16	12
29	17	5
30	19	5
1964		
17 Feb.	27	14
10 Apr.	38	1
10	40	1
11	42	22
12	46	32
13	49	5
8 June	71	4
9	77	3
11	83	1
12	85	17
13	89	3
13	92	1
7 July	93	28
8	94	21
9	95	27
10	96	6
12	98	18
18 Sept.	118	10
20	119	12
22	120	9
22	122	16
23	123	7
24	127	2
25	128	5
26	134	8
27	138	18

Total: 39 transits 393 simultaneous points

The contribution of tracking stations to the observational material is the following:

TABLE 2

	Country	Station number	Number of fixes
Ryazan	U.S.S.R.	1042	136
Baja	Hungary	1113	124
Budapest	Hungary	1111	84
Vologda	U.S.S.R.	1014	84
Arkhangelsk	U.S.S.R.	1004	80
Cluj	Roumania	1132	47
Chernovtsy	U.S.S.R.	1062	45
Riga	U.S.S.R.	1040	36
Kiev	U.S.S.R.	1023	29
Kishinev	U.S.S.R.	1024	27
Bautzen	G.D.R.	1120	21

TABLE 2 (cont.)

	Country	Station number	Number of fixes
Tartu	U.S.S.R.	1051	17
Olsztyn	Poland	1151	17
Rostov	U.S.S.R.	1041	14
Rodewisch	G.D.R.	1185	8
Erevan	U.S.S.R.	1018	7
Dnepropetrovsk	U.S.S.R.	1017	5
Stara Zagora	Bulgaria	1102	5
Birsk	U.S.S.R.	1085	4
Kazan	U.S.S.R.	1020	1
Petrozavodsk	U.S.S.R.	1038	1
Zvenigorod	U.S.S.R.	1072	1

Total: 22 stations 793 fixes.

Plotting the celestial coordinates against time, pairs of simultaneous observations have been selected by the aforementioned method of graphic interpolation. If serial fixes were made from several stations at the same time, we tried to adopt the procedure of interpolation on the sequences of each station. Single observations are weighted by *1*; coordinates interpolated at the beginning or at the end of a sequence (uncertain values) by *2*; reliably interpolated values by *3*.

The authors would like to thank Mr M. Ill and Mr K. Sütő for kindly permitting the use of pairs of simultaneous points selected by them from the material of the Aug. 1963, Feb. and Apr. 1964 intervals.

THE METHOD

Instantaneous positions in space of the satellite have been determined from pairs of simultaneous fixes according to the procedure proposed at the Riga conference [9]. Its program used in the computer reduction (electronic computer Type Elliott 803) is given in the Appendix.

Starting from observations in the equatorial coordinates ($\alpha_1\delta_1$ and $\alpha_2\delta_2$ respectively) equations are written for planes in a rectangular system fixed in space, satisfying the following two conditions: a) they each contain one observing station, M_1 or M_2; b) they contain angles α_1 and α_2 with the $[XY]$ plane of the coordinate system respectively.

The coordinates x_s and y_s of the satellite are given by the line of intersection of the planes (μ). If straight lines are drawn inside these planes containing the angles $90° - \delta_1$, $90° - \delta_2$ (measured from the North Pole) from stations M_1 and M_2 respectively, their intersection with μ provides two fictitious positions of the satellite in space ($S_{\rm I}$, $S_{\rm II}$) (see Fig. 1). Forming their appropriately weighted mean value, points in space can be obtained which

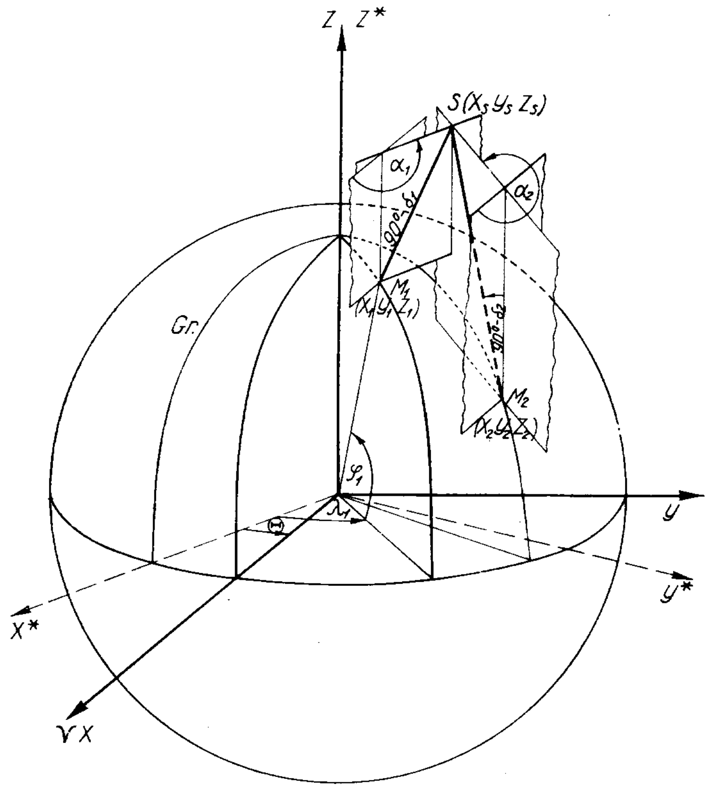


Fig. 1.

serve as a basis for further calculations (S_1, S_2, . . .). The set of equations used is a simple consequence of the basic equation of cosmic triangulation [10]:

$$x_s = \frac{y_2 - y_1 - x_2 \operatorname{tg} \alpha_2 + x_1 \operatorname{tg} \alpha_1}{\operatorname{tg} \alpha_1 - \operatorname{tg} \alpha_2}$$

$$y_s = y_1 + (x_s - x_1) \operatorname{tg} \alpha_1$$

$$z_s = z_1 (x_s - x_1) \operatorname{tg} \delta_1 \sec \alpha_1 = z_2 + (x_s - x_2) \operatorname{tg} \delta_2 \sec \alpha_2$$

It is to be noted that the method offers the advantage of providing a pair of solutions even in the case of not absolutely accurate observations (without a least-squares solution), as the errors in the initial data present themselves in the deviation of the two different results for the z_s coordinate of the satellite. The use of serial fixes makes a graphic smoothing of the x_s, y_s, z_s and $R_s = \sqrt{x_s^2 + y_s^2 + z_s^2}$ values feasible.

I. D. Zhongolovich proposed in a paper presented at the Riga conference under the title "Remarks to the Interobs program" that changes of the nodal period of revolution of satellites should be determined from the visual observations. He proved that sequences of spatial positions give transit times above a given geographic latitude circle (φ_0) accurate to at least a few tenths of a second. Consequently from two passes observed on two consecutive nights the mean period of revolution of the satellite is determinable to a few hundredths of a second. There is hereby a hopeful possibility to analyse quick period changes of low-orbit satellites with an adequate accuracy.

First of all the geocentric latitude of the subsatellite points (φ_n) is needed. It can be easily derived from space coordinates, namely through

$$\sin \varphi_n = \frac{z_n}{R_n}.$$

Plotting φ_n as a function of the epoch of observation (t_n) we are able to perform on the smoothed curve a graphic determination of the satellite's crossing time over the given latitude circle (t_0). The numerical procedure proposed by Zhongolovich is more accurate and makes extrapolation possible. If the path element in question is short, equation

$$t_0 = t_n - \left(\frac{R_n}{\varkappa}\right)^2 (\mu_n - \mu_0) \qquad \sin \mu_n = \frac{\sin \varphi_n}{\sin i}$$

holds true, where

$$\sin \mu_0 = \frac{\sin \varphi_0}{\sin i} \qquad \varkappa = 190.2 \sqrt[4]{a(1 - e^2)}$$

if R_n and a are expressed in kilometers, $(t_n - t_0)$ in seconds and $(\mu_n - \mu_0$ in degrees.

Calculating the transit times t_0 belonging to φ_0 from every pair $(t_n\, \varphi_n)$ separately, their mean value $\bar{t}_0$ is to be formed. As a first step, however, approximate values of i, a and e are needed. Orbital elements published by prediction services might be used as a first approximation. Taking them as a starting point, all t_0 values on a selected long transit of high quality should be calculated. If the t_0 values thus obtained show a monotonous decrease or increase as a function of time, the adopted value of i has to be corrected. To derive its correct value we may proceed as follows: if e.g. $t_{01} < t_0 < t_{02}$ where t_{01} and t_{02} are arbitrary t_0 values at the beginning (t_{n1}) and at the end (t_{n2}) of the given transit respectively, we have

$$(t_n - \bar{t}_0)' = k(t_n - \bar{t}_0) \quad \text{where} \quad k = 1 + \frac{t_{02} - t_{01}}{t_{n2} - t_{n1}}$$

$$(\mu_n - \mu_0)' = \frac{k}{\bar{R}_n^2} (\mu_n - \mu_0)$$

$$\sin i' = \frac{\sin^2 \varphi_0 - 2 \sin \varphi_0 \sin \varphi_n \cos (\mu_n - \mu_0)' + \sin^2 \varphi_n}{\sin (\mu_n - \mu_0)'}.$$

Dashed letters are the corrected quantities. A mean value of i', derived from several pairs of (t_{n1}, t_{n2}) equals the correct inclination of the orbital plane.

Then follows the determination of a using $\bar{t}_0$ values of a pair of well observed transits separated by nearly 24 hours ($\bar{t}_{0A}$, $\bar{t}_{0B}$). With the approximate value of the orbital period, the revolution number (n) between the two selected transits can be guessed. Hence we get

$$P' = \frac{\bar{t}_{0A} - \bar{t}_{0B}}{n}$$

the nodal period of revolution in a closer approximation. Kepler's third law then yields a (the deviation between nodal and sidereal period of revolution is not significant — see later) and combining it with e we obtain $\varkappa$. The value of e can be easily estimated if there are observations near perigee or apogee (i.e. R_{per} or R_{ap} is known), because then

$$e = \frac{R_{\mathrm{ap}}}{a} - 1 = 1 - \frac{R_{\mathrm{per}}}{a}$$

will hold.

Finally, using the corrected i and $\varkappa$ values, the calculation of t_0 from every point of every transit has to be completed. It is advisable to choose φ_0 inside or at least near to the observed portions of the orbits in question, but experience shows that it is acceptable also if subsatellite points are not farther from the selected latitude circle than 6—8 degrees. The mean value of the t_0 values represents the observed crossing time over the latitude φ_0 during a given transit:

$$\bar{t}_{0n} = O_n$$

For organizational reasons, as a rule, Interobs cycles lasted 7—10 days per month, therefore period changes were investigated within each cycle separately.

Calculations may be carried out as follows. Let us suppose as a first approximation that the period is constant and equals to its approximate value, P', furthermore that the starting epoch

$$C_0 = O_0 \text{ and}$$

$$C_n = C_0 + nP' \text{ where } n = 0, 1, 2 \ldots$$

Plotting the $O - C$ values against time, usually a parabola is obtained, in accordance with the general tendency of decreasing of the period of revolution. Then the P_0 period, belonging to the first of the transits observed, is to be determined, using either graphic or — following a proposal of M. Ill — numerical solution. He supposes that during the given time interval nodal period increases steadily by Δ sec per each revolution, then

$$P_0 = P' + \delta$$

where

$$\delta = \frac{n-1}{k(n-k)}(O-C)_k - \frac{k-1}{n(n-k)}(O-C)_n$$

and the period changes during one revolution by

$$\Delta_n = \frac{2(O-C)_n}{n(n-k)} - \frac{2(O-C)_k}{k(n-k)}$$

where $(O - C)_k$ belongs to the k^{th}, $(O - C)_n$ to the n^{th} orbit and $n \neq 0$ $k \neq 0$ $n > k$. For practical purposes two well observed transits should be selected.

Repeating the calculation of residuals using the P_0 value from either the graphic or the numerical solution we have

$$C_0 = O_0 \qquad C_n = C_0 + nP_0$$

which yields as a result the rate of change of the nodal period for each transit separately

$$\Delta_n = \frac{dP}{dn} = \frac{2(O - C)_n}{n(n - 1)}.$$

If these results are equal, i.e. $\Delta_n = \text{const.}$, our assumption appears to be valid and the procedure yields the period change in the whole interval in question. If Δ_n varies, we should try to divide the interval into several parts and select points near enough to each other to allow the determination of a $\Delta_n = \text{const.}$ value. In practice it may seem reasonable to plot first $2(O - C)_n$ as a function of $n(n - 1)$ and test whether the points lie on a single straight line or not. If it can be assumed that $\Delta_n = \text{const.}$, it is worth while to carry out the inverse procedure, namely to determine P_z, the period of revolution belonging to the last transit of the interval. Then in the opposite direction

$$C_z = O_z \qquad C_m = C_z - mP_z \quad \text{where } m = 0, 1, 2 \ldots$$

hence Δ_m can be calculated exactly in the same manner as in the previous case.

The influence of the errors of the observed O_n values on Δ_n being inversly proportional to $n(n - 1)$, we formed a weighted mean of the Δ_n and Δ_m pairs for every transit separately

$$\bar{\Delta} = \frac{n(n - 1)\,\Delta_n + m(m - 1)\,\Delta_m}{n(n - 1) + m(m - 1)}.$$

The result has been considered as the rate of decrease of the period (in sec/rev) during the time interval under consideration.

RESULTS

The part of the observational material published in [7], seemingly most suitable to an investigation of the type outlined above, consisted of 39 transits of 1960 $\varepsilon 3$. Altogether 6 transits have been rejected, partly because the path element observed was too far from the latitude circle φ_0 (pass No. 5), partly because it belonged to a time interval where the number of suitable transits to deduce period changes proved to be insufficient (pass No. 27 and passes No. 77, 83, 89, 92 respectively).

One third of all positions in space, given by the computer program, have been similarly neglected (126 points out of 393). These points have been rejected because

a) they belong to transits already cancelled;
b) they were far from the latitude circle φ_0;

c) the length of their radius vector, R, differed considerably from all other R values of the transit (in some cases it is a consequence of the decreased parallax of the satellite as seen from the pair of stations);

d) another set of $XYZR$ values could be derived from other observations but referring to the same moment. In this case corresponding quantities have been averaged.

A review of the whole observational material is given in Table 1.

Positions in space actually used, the partial results of the Zhongolovich method and transit times are summarized in Table 3.

TABLE 3

	t_n h	t_n m	t_n s	z_n km	R_n km	$t_0 - t_n$ s	t_0 h	t_0 m	t_0 s
1963									
21.8	20	34	2.8	6020.	6858.83	20.1	20	34	22.9
		34	6.7	6010.5	6848.37	19.9		34	26.6
22.8	21	4	17.9	6110.43	6859.57	81.2	21	5	39.1
		4	31.4	6090.99	6861.64	64.9		5	36.3
		4	36.4	6087.69	6861.62	62.5		5	38.9
		4	40.2	6082.55	6861.98	58.7		5	38.9
		5	3.7	6052.	6863.	37.6		5	41.3
		5	5.3	6048.	6864.	34.5		5	39.8
		5	27.2	6015.09	6862.84	15.3		5	42.5
		6	5.0	5942.	6866.	− 25.1		5	39.9
		6	24.0	5902.50	6867.69	− 44.7		5	39.3
		6	34.2	5878.51	6866.34	− 55.1		5	39.1
		6	44.0	5853.54	6862.84	− 64.7		5	39.3
23.8	20	3	43.9	6094.75	6857.87	70.2	20	4	54.1
		3	51.9	6082.94	6855.64	62.9		4	54.8
		4	3.1	6062.33	6856.60	47.9		4	51.0
		4	5.8	6060.84	6855.22	47.7		4	53.5
		4	30.0	6022.95	6857.82	22.4		4	52.4
		4	43.4	6001.38	6860.18	8.8		4	52.2
		4	58.7	5974.39	6859.39	− 5.6		4	53.1
		5	5.0	5970.77	6874.76	− 10.0		4	55.0
		5	46.5	5874.21	6858.28	− 53.8		4	52.7
		6	5.0	5833.03	6859.42	− 72.2		4	52.8
24.8	19	4	0.2	5741.15	6588.34	3.7	19	4	3.9
	20	34	57.6	6083.11	6851.52	65.6	20	36	3.2
		35	15.1	6058.43	6853.67	47.0		36	2.1
		35	17.8	6054.62	6851.09	46.0		36	3.8
		35	38.5	6024.46	6856.65	23.9		36	2.4
		36	29.0	5932.74	6858.58	− 26.5		36	2.5
25.8	19	34	4.3	6103.25	6865.52	71.6	19	35	15.9
		36	28.6	5831.57	6879.74	− 80.5		35	8.1
26.8	20	5	26.8	6062.73	6859.66	46.4	20	6	13.2
		5	30.4	6058.48	6858.20	44.5		6	14.9
		5	32.7	6055.18	6857.24	42.8		6	15.5
		5	33.3	6054.48	6857.11	42.5		6	15.8
		5	40.4	6044.36	6855.46	36.9		6	17.3
		6	0.4	6012.86	6856.22	17.3		6	17.7
		7	39.0	5808.94	6855.04	− 80.6		6	18.4
27.8	19	5	14.4	6000.01	6855.34	10.4	19	5	24.8
		5	52.1	5938.	6858.99	− 24.1		5	28.0
		6	4.0	5912.84	6874.50	− 42.7		5	21.3

TABLE 3 (cont.)

	t_n h	t_n m	t_n s	z_n km	R_n km	$t_o - t_n$ s	t_o h	t_o m	t_o s
		6	5.0	5910.95	6875.98	— 44.2		5	20.8
		6	39.3	5825.51	6857.24	— 74.5		5	24.8
	20	36	49.8	6092.35	6928.63	27.3	20	37	17.1
		39	3.7	5760.66	6860.36	—101.9		37	21.8
		39	4.6	5754.54	6854.32	—102.1		37	22.5
		39	5.6	5747.28	6844.68	—101.5		37	24.1
29.8	18	34	32.3	6073.52	6855.59	56.2	18	35	28.5
		34	59.1	6032.15	6857.14	28.3		35	27.4
		35	3.8	6029.80	6860.94	24.9		35	28.7
		35	4.6	6031.07	6863.07	24.6		35	29.2
		35	5.5	6032.27	6865.09	24.2		35	29.7
		36	3.8	5905.21	6851.47	— 36.7		35	27.1
		36	4.8	5903.23	6851.77	— 37.8		35	27.0
		36	6.1	5901.27	6852.49	— 39.0		35	27.1
		36	10.5	5900.06	6859.18	— 42.4		35	28.1
	20	7	23.4	5990.36	6862.02	1.8	20	7	25.2
		7	49.8	5939.31	6863.97	— 25.6		7	24.2
		7	52.9	5941.68	6871.21	— 27.6		7	25.3
30.8	19	6	34.6	5968.99	6859.71	— 8.6	19	6	26.0
		6	34.8	5969.20	6859.39	— 8.3		6	25.5
1964									
10.4	18	14	7.5	5107.44	6753.54	10.6	18	14	18.1
	19	46	14.2	5287.86	6780.66	— 32.9	19	45	41.3
11.4	18	34	37.2	4888.92	6822.06	78.5	18	35	55.7
		34	40.8	4902.36	6817.63	74.4		35	55.2
		34	43.3	4910.40	6813.19	71.6		35	54.9
		34	44.9	4916.75	6812.34	69.9		35	54.8
		34	48.1	4927.56	6806.41	66.2		35	54.3
		34	51.0	4940.47	6806.28	63.0		35	54.0
		34	52.5	4944.53	6803.25	61.5		35	54.0
		34	56.3	4963.	6801.32	56.6		35	52.9
		34	59.0	4974.	6795.90	52.8		35	51.8
		34	59.6	4976.64	6805.79	54.1		35	53.7
		35	2.9	4994.18	6810.19	50.7		35	53.6
		35	5.9	5009.50	6813.21	47.6		35	53.5
		35	9.8	5027.16	6814.47	43.3		35	53.1
		35	11.5	5032.33	6812.09	41.5		35	53.0
		35	13.1	5037.87	6810.68	39.9		35	53.0
		35	15.8	5047.97	6809.14	37.0		35	52.8
		35	18.4	5056.	6802.86	33.7		35	52.1
		35	20.8	5059.60	6799.49	32.2		35	53.0
		35	24.9	5072.70	6795.92	28.1		35	53.0
12.4	18	57	18.0	5143.25	6781.82	6.9	18	57	24.9
		57	23.5	5175.94	6790.47	0.0		57	23.5
		57	28.7	5195.29	6790.15	— 5.3		57	23.4
		57	31.7	5201.69	6786.30	— 7.8		57	23.9
		57	33.9	5207.49	6786.19	— 9.4		57	24.5
		57	39.0	5218.91	6779.86	— 13.8		57	25.2
		57	43.5	5223.77	6770.78	— 17.0		57	26.5
		57	46.3	5253.41	6785.25	— 22.2		57	24.1
		57	50.3	5262.78	6780.38	— 25.9		57	24.4
		57	55.3	5288.62	6786.64	— 31.8		57	23.5
		57	59.4	5299.50	6784.09	— 35.5		57	23.9
		58	4.6	5312.73	6780.55	— 40.0		57	24.6

TABLE 3 (cont.)

	t_n h	m	s	z_n km	R_n km	$t_o - t_n$ s	t_o h	m	s
		58	9.4	5325.72	6777.97	— 44.3		57	25.1
		58	14.0	5340.78	6777.23	— 48.9		57	25.1
		58	18.8	5358.97	6778.01	— 54.0		57	24.8
		58	23.4	5376.34	6778.87	— 59.0		57	24.4
		58	26.9	5387.13	6778.15	— 62.4		57	24.5
		58	30.8	5400.01	6777.84	— 66.4		57	24.4
		58	33.8	5408.82	6777.49	— 69.2		57	24.6
		58	38.8	5414.63	6770.89	— 72.4		57	26.4
		58	42.8	5430.92	6772.68	— 77.0		57	25.8
13.4	17	48	47.6	5426.59	6775.45	10.2	17	48	57.8
		48	49.3	5437.17	6775.39	6.9		48	56.2
		48	56.8	5458.46	6776.53	0.5		48	57.3
		50	14.1	5679.95	6774.16	— 76.6		48	57.5
8.6	0	18	36.2	4643.84	6754.14	102.3	0	20	18.5
		18	48.4	4697.44	6752.85	90.1		20	18.5
		19	1.9	4759.26	6751.10	75.8		20	17.7
		19	17.6	4820.83	6747.15	60.8		20	18.4
12.6	0	5	42.0	4933.49	6758.85	36.1	0	6	18.1
		6	16.5	5069.15	6749.97	0.4		6	16.9
		6	19.1	5069.95	6742.04	— 1.3		6	17.8
		6	20.5	5084.84	6750.54	— 3.5		6	17.0
		6	24.6	5100.91	6750.15	— 7.8		6	16.8
		6	31.2	5125.03	6748.76	— 14.4		6	16.8
		6	33.1	5133.45	6749.98	— 16.3		6	16.8
		6	34.6	5125.46	6739.29	— 16.3		6	18.3
		6	37.6	5134.63	6737.21	— 19.2		6	18.4
		6	42.6	5143.52	6729.61	— 23.0		6	19.6
		6	44.7	5175.52	6747.64	— 28.1		6	16.6
		7	13.4	5276.78	6743.11	— 57.1		6	16.3
		7	19.2	5287.78	6736.80	— 61.5		6	17.7
7.7	0	37	59.6	4860.98	6601.83	— 56.2	0	37	3.4
		38	0.2	4854.41	6601.74	— 55.3		37	4.9
		38	3.8	4864.64	6635.36	— 59.4		37	4.4
		38	3.9	4866.70	6640.38	— 59.9		37	4.0
		38	7.3	4852.35	6639.35	— 63.1		37	4.2
		38	10.3	4843.32	6644.37	— 66.2		37	4.1
		38	12.3	4831.54	6639.01	— 68.0		37	4.3
		38	16.6	4814.69	6640.10	— 72.2		37	4.4
		38	18.4	4809.48	6643.77	— 74.1		37	4.3
		38	20.4	4800.52	6642.09	— 75.9		37	4.5
		38	22.9	4791.36	6645.93	— 78.8		37	4.1
		38	24.2	4785.95	6645.32	— 80.0		37	4.2
		38	26.5	4777.49	6648.37	— 82.6		37	3.9
		38	28.6	4768.32	6649.01	— 84.8		37	3.8
		38	33.5	4744.89	6642.83	— 89.0		37	4.5
		38	34.9	4738.86	6644.81	— 90.8		37	4.1
		38	42.1	4706.07	6639.95	— 97.3		37	4.8
		38	44.2	4697.43	6640.31	— 99.3		37	4.9
		38	47.1	4684.79	6639.51	—102.1		37	5.0
		38	47.8	4680.92	6638.97	—102.9		37	4.9
		38	50.5	4668.94	6637.18	—105.2		37	5.3
		38	56.6	4641.99	6638.00	—111.4		37	5.2
		38	59.1	4632.78	6636.94	—113.3		37	5.8
		39	1.3	4621.96	6637.93	—115.8		37	5.5

TABLE 3 (cont.)

	t_n h	t_n m	t_n s	z_n km	R_n km	$t_0 - t_n$ s	t_0 h	t_0 m	t_0 s
		39	6.9	4599.51	6636.37	−120.5		37	6.4
8.7	23	42	45.9	5128.98	6652.95	4.4	23	42	50.3
		42	58.8	5067.39	6643.29	−10.0		42	48.8
		42	58.9	5073.59	6647.58	−9.3		42	49.6
		42	59.6	5072.33	6649.09	−9.9		42	49.7
		43	1.3	5055.44	6641.69	−12.8		42	48.5
		43	1.6	5063.18	6648.40	−12.1		42	49.5
		43	2.8	5068.83	6656.49	−12.3		42	50.5
		43	3.3	5056.21	6648.41	−14.0		42	49.3
		43	3.8	5043.74	6640.33	−15.6		42	48.2
		43	4.5	5054.12	6650.77	−15.0		42	49.5
		43	7.3	5019.17	6632.89	−20.4		42	46.9
		43	12.0	5000.94	6631.83	−24.9		42	47.1
		43	13.3	5004.66	6638.05	−25.2		42	48.1
		43	16.8	5003.09	6647.08	−27.4		42	49.4
		43	19.0	4998.61	6650.34	−29.2		42	49.8
9.7	22	29	17.5	5277.51	6654.29	45.9	22	30	3.4
		29	30.8	5221.53	6643.98	32.0		30	2.8
		29	31.1	5229.64	6650.71	31.4		30	2.5
		29	32.8	5217.71	6643.07	31.1		30	3.9
		29	33.7	5223.84	6650.89	31.1		30	4.8
		29	35.5	5217.37	6650.22	29.4		30	4.9
		29	47.8	5172.63	6649.83	17.0		30	4.8
		29	48.8	5170.49	6649.04	16.6		30	5.4
		30	1.5	5124.09	6649.02	3.9		30	5.4
		30	1.8	5120.42	6647.84	3.2		30	4.6
		30	2.9	5115.17	6648.13	1.7		30	4.6
		30	3.5	5113.63	6647.84	1.3		30	4.8
		30	4.2	5110.01	6649.05	0.1		30	4.3
		30	4.3	5111.41	6651.43	0.0		30	4.3
		30	5.6	5103.68	6648.23	−1.4		30	4.2
		30	11.0	5081.84	6644.50	−6.4		30	4.6
		30	12.0	5072.41	6640.64	−8.1		30	3.9
		30	17.7	5063.50	6649.84	−12.4		30	5.3
		30	17.9	5062.39	6648.28	−12.3		30	5.6
10.7	22	48	3.4	5187.55	6638.88	23.5	22	48	26.9
		48	4.3	5179.83	6634.82	22.1		48	26.4
		48	7.4	5174.19	6636.66	20.2		48	27.6
		48	15.6	5163.91	6653.04	13.9		48	29.5
		48	16.3	5152.48	6647.66	11.9		48	28.2
		48	17.3	5137.48	6641.20	9.2		48	26.5
12.7	21	52	27.8	5427.84	6649.54	93.5	21	54	1.3
		52	28.8	5425.39	6649.57	92.7		54	1.5
		52	29.9	5421.51	6649.09	91.5		54	1.4
		52	30.8	5419.21	6649.34	90.7		54	1.5
		52	32.0	5415.74	6649.30	89.6		54	1.6
		52	32.7	5413.68	6649.39	88.9		54	1.6
		52	33.7	5411.67	6659.71	85.8		53	59.5
		53	14.1	5281.86	6645.06	49.2		54	3.3
		53	19.4	5257.84	6653.90	40.4		53	59.8
		53	19.8	5262.27	6647.48	42.9		54	2.7
		53	31.1	5211.77	6649.45	27.9		53	59.0
		53	32.4	5205.10	6649.63	26.1		53	58.5
		53	33.6	5208.04	6648.43	27.2		54	0.8
18.9	2	5	0.6	4954.33	6644.91	−933.9	1	49	26.7

TABLE 3 (cont.)

	t_n h	m	s	z_n km	R_n km	$t_o - t_n$ s	t_o h	m	s
		5	2.3	4948.46	6643.43	—934.6		49	27.7
		5	29.7	4829.03	6636.74	—960.6		49	29.1
		5	32.2	4813.30	6626.59	—959.6		49	32.6
		5	34.1	4812.87	6640.79	—966.3		49	27.8
		5	45.2	4760.84	6630.23	—973.6		49	31.6
		6	5.7	4675.89	6636.61	—996.0		49	29.7
		6	11.2	4657.66	6636.95	—1000.2		49	31.0
20.9	17	30	6.1	4926.57	6674.28	112.0	17	31	58.1
		30	7.8	4930.97	6671.76	110.4		31	58.2
		30	9.7	4938.37	6671.38	108.4		31	58.1
		30	19.8	4982.93	6673.96	97.8		31	57.6
		30	26.5	4989.78	6658.22	92.7		31	59.2
		30	30.7	5031.35	6678.74	86.6		31	57.3
		30	49.3	5095.04	6671.35	68.4		31	57.7
		30	50.3	5096.26	6669.22	67.6		31	57.9
22.9	16	34	2.6	5477.57	6656.56	— 48.7	16	33	13.9
		34	3.8	5479.62	6655.14	— 49.9		33	13.9
		34	4.8	5482.16	6654.74	— 50.8		33	14.0
		34	5.8	5485.12	6654.91	— 51.8		33	14.0
		34	6.9	5487.38	6653.99	— 52.5		33	14.4
		34	7.9	5491.00	6654.71	— 53.8		33	14.1
		34	35.6	5568.86	6655.83	— 81.3		33	14.3
		34	36.8	5573.55	6657.15	— 82.6		33	14.2
		34	38.0	5578.89	6658.95	— 84.1		33	13.9
	18	3	29.6	5178.92	6668.67	45.1	18	4	14.7
		4	2.9	5297.01	6667.70	11.1		4	14.0
		4	30.5	5391.38	6668.67	— 17.5		4	13.0
		4	48.8	5442.07	6664.26	— 34.9		4	13.9
		4	52.0	5447.39	6664.68	— 36.5		4	15.5
		4	59.5	5471.38	6662.69	— 45.0		4	14.5
		5	4.6	5493.56	6666.04	— 51.7		4	12.9
		5	30.4	5542.81	6649.05	— 73.7		4	16.7
23.9	18	21	34.1	5521.20	6642.81	— 67.6	18	20	26.5
		22	3.2	5574.84	6631.69	— 90.5		20	32.7
		22	29.5	5618.38	6620.82	—110.8		20	38.7
		22	31.3	5700.50	6661.52	—132.1		20	19.2
		22	32.5	5700.74	6660.27	—132.6		20	19.9
		23	1.0	5823.06	6695.36	—177.3		20	3.7
24.9	18	38	2.4	5608.90	6655.49	— 96.5	18	36	25.9
		38	34.2	5677.66	6649.56	—126.2		36	28.0
25.9	17	23	1.7	5609.73	6656.91	— 96.4	17	21	25.3
		23	3.1	5612.12	6655.79	— 97.7		21	25.4
		23	17.9	5644.91	6650.97	—112.2		21	25.7
		23	18.6	5644.74	6648.72	—112.8		21	25.8
		23	19.0	5644.56	6647.10	—113.2		21	25.8
26.9	17	39	0.2	5618.85	6667.50	— 96.8	17	37	23.4
		40	2.3	5742.56	6651.61	—154.4		37	27.9
		40	2.7	5742.10	6651.25	—154.3		37	28.4
		40	3.1	5743.18	6651.82	—154.6		37	28.5
		40	3.6	5746.71	6653.99	—155.5		37	28.1
27.9	16	25	3.7	5764.70	6657.87	—162.6	16	22	21.1
		25	4.7	5765.68	6656.70	—163.5		22	21.2
		25	5.7	5766.35	6655.23	—164.4		22	21.3
		25	6.5	5767.83	6654.72	—165.3		22	21.2
		25	32.9	5805.78	6647.18	—187.6		22	25.3

TABLE 3 (cont.)

t_n h	m	s	z_n km	R_n km	$t_o - t_n$ s	t_v h	m	s
	25	33.2	5796.57	6640.98	—185.2	22	28.0	
	25	33.9	5796.86	6640.72	—185.5	22	28.4	
	25	35.0	5800.53	6642.86	—186.5	22	28.5	
	25	35.8	5798.89	6641.80	—186.1	22	29.7	
	25	36.9	5805.03	6645.17	—188.0	22	28.9	
	26	4.2	5858.08	6647.43	—217.2	22	27.0	
	26	5.1	5859.95	6648.00	—218.1	22	27.0	
	26	6.0	5860.88	6647.78	—218.8	22	27.2	
	26	6.9	5861.11	6646.86	—219.4	22	27.5	

Different kinds of Δ values deduced from 31 passes by graphic or numerical solution are given in Table 4. In the first column "O" means simply the mean of the t_0 values of Table 3, averaged for each revolution separately. In the second column the root-mean-square error of $\bar{t}_0$ (expressed in seconds) is given for all transits which include a large enough number of observations. Its average value is 0.656.

Note: The epoch $O = 496.259943$ comes from two O values combined from two consecutive transits, each containing one pair of simultaneous observations only.

The next table gives the period at the beginning and at the end of each observation cycle, its mean value, the mean rate of decrease of the period during each time interval, and its standard error. (As $\frac{dP}{dn}$ changed suddenly on 26 August 1963, the first cycle has been divided into two parts.) Then follows, as a comparison, $\left(\frac{dP}{dn}\right)^*$, defined as the slope of a plot of $\bar{P}$ against t at the cycles of observation. Finally theoratical values of $\frac{dP}{dn}$ are given $\left(\left(\frac{dP}{dn}\right)_{t1} \text{ and } \left(\frac{dP}{dn}\right)_{t2}\right)$ calculated from the following equations

$$\left(\frac{dP}{dn}\right)_{t1} = -\frac{3}{4}\frac{e_0}{t_L}\frac{T_0}{(1-t/t_L)^{1/2}}$$

$$\left(\frac{dP}{dn}\right)_{t2} = -\frac{9}{10}\frac{e_0}{t_L}\frac{T_0}{(1-t/t)^{1/2}}$$

[11] where $T_0 = 94.27$ min., $e_0 = 0.030$ are initial orbital elements of the satel-

TABLE 4

O JD 2438... day	σ sec	Weight	$(O-C)_n$ day	$n(n-1)$	Δ_n sec/rev	$(O-C)_m$ day	$m(m-1)$	Δ_m sec/rev	$\bar{\Delta}$ sec/rev
263.357229	± —	1	—	0	—	—.000292	4556	—.0111	—.0111
264.378929	.467	10	—.000012	240	—.0086	—.000161	2652	—.0105	—.0103
265.336725	.389	8	—.000071	930	—.0132	—.000086	1332	—.0112	—.0120
266.294488	—	0.5	—.000162	2070	—.0135	—.000044	462	—.0165	—.0140
266.358366	.309	5	—.000141	2162	—.0113	—.000014	420	—.0058	—.0104
267.316111	—	1	—.000251	3782	—.0115	+.000009	30	+.0518	—.0115
268.337686	.683	7	—.000004	110	—.0063	—.000177	3782	—.0081	—.0080
269.295416	1.320	5	—.000024	650	—.0064	—.000092	2256	—.0070	—.0069
269.359276	—	4	—.000014	702	—.0035	—.000075	2162	—.0060	—.0054
271.274631	.333	8	—.000159	3192	—.0086	—.000010	272	—.0063	—.0084
271.338481	—	3	—.000159	3306	—.0083	—.000003	240	—.0021	—.0079
272.296129	—	1	—.000261	5256	—.0086	—	0	—	—.0086
496.259943	—	1	—	0	—	—.000168	2162	—.0134	—.0134
497.274926	.237	8	—.000019	240	—.0137	—.000069	930	—.0128	—.0130
498.289868	.186	8	—.000079	992	—.0138	—.000012	210	—.0099	—.0131
499.241334	—	3	—.000178	2162	—.0142	—	0	—	—.0142
583.525747	.136	10	—	0	—	—.000275	8556	—.0056	—.0056
585.488068	.276	8	—.000027	930	—.0050	—.000117	3782	—.0053	—.0053
586.437551	.194	9	—.000067	2070	—.0056	—.000068	2162	—.0054	—.0055
587.450319	.486	4	—.000123	3782	—.0056	—.000029	930	—.0054	—.0056
589.412511	.389	8	—.000279	8556	—.0056	—	0	—	—.0056
656.576036	.582	3	—	0	—	—.000781	22952	—.0059	—.0059
659.230532	.200	8	—.000068	1722	—.0068	—.000416	11990	—.0060	—.0061
661.189746	.197	9	—.000175	5256	—.0058	—.000203	6162	—.0057	—.0057
661.252945	.448	5	—.000180	5402	—.0058	—.000198	6006	—.0057	—.0057
662.264160	4.994	0.5	—.000227	8010	—.0049	—.000080	3782	—.0037	—.0045
663.275312	—	1	—.000338	11130	—.0053	—.000026	2070	—.0022	—.0048
664.223213	.105	5	—.000496	14520	—.0059	—.000029	930	—.0054	—.0059
665.234344	.971	4	—.000627	18632	—.0058	—.000005	210	—.0041	—.0058
666.182243	.865	3	—.000787	22952	—.0059	—	0	—	—.0059

TABLE 5

Date	P_0 day	P_z day	$\bar{P}$ day	$\frac{dP}{dn}$ sec	σ	$\left(\frac{dP}{dn}\right)^*$ sec	$\left(\frac{dP}{dn}\right)_{t1}$ sec	$\left(\frac{dP}{dn}\right)_{t2}$ sec
Aug 1963 I.	.0638570	.0638443	.0638506	—.0110	±.0006	—.0101	—.0072	—.0084
Aug 1963 II.	.0638570	.0638443	.0638506	—.00755	.00049	—.0101	—.0072	—.0084
Apr 1964	.0634376	.0634303	.0634340	—.0132	.0003	—.0092	—.0089	—.0103
Jun 1964	—	—	.0633376	—	—	—	—	—
Jul 1964	.0633015	.0632956	.0632986	—.00550	.00006	—.0075	—.0098	—.0113
Sep 1964	.0632039	.0631936	.0631987	—.00582	.00020	—.0068	—.0109	—.0126

TABLE 6

Date	i	e	$\varkappa$	φ_0	μ_0	U_s min	a km	h_{ap} km	h_{per} km
Aug 1963	64.94	.0173	1723.73	60.750	74.394	92.001	6750.80	490	256
Apr 1964	64.98	.0152	1721.87	49.662	57.264	91.401	6721.45	445	241
June 1964	64.98	.011	1721.34	—	—	91.264	6714.70	411	263
July 1964	64.98	.011	1721.32	50.217	58.000	91.208	6711.96	408	260
Sept 1964	64.97	.0104	1720.85	53.131	61.996	91.064	6704.89	397	256

lite, and t_L its total lifetime. Orbital elements in the predictions for March 1965, using the inverse form of the equations quoted above, yielded lifetimes 1874 days and 1896 days respectively. (The actual lifetime of the satellite proved to be somewhat longer.)

Figures No 2—8 have been plotted to illustrate the graphic solution of the problem. On Fig. No 2, 4, 5 and 7 deviations between observed and

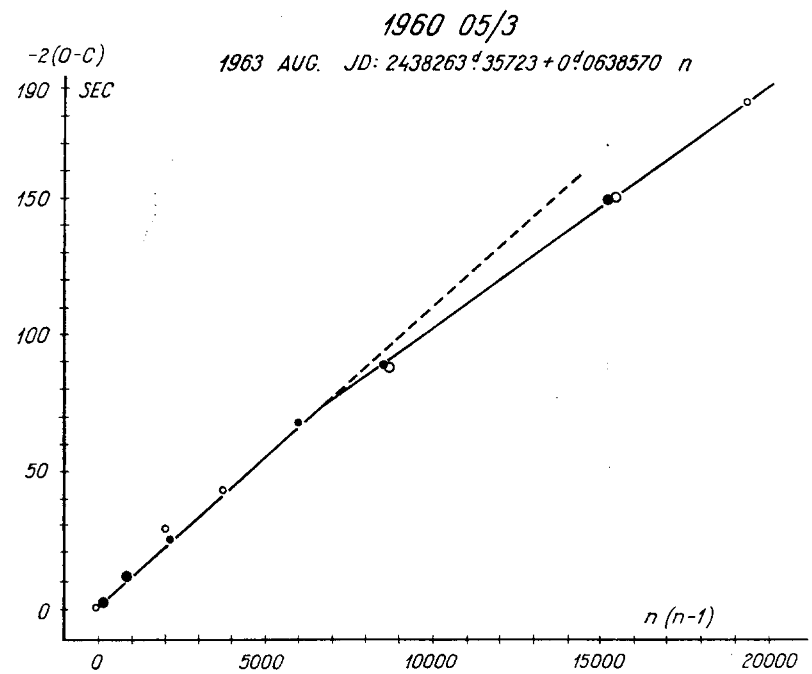


Fig. 2.

calculated transit times $(O - C)_n$ are plotted against $n(n - 1)$; the slope of each straight line gives Δ_n. C_n values have been derived using the initial period of revolution P_0 and the starting date O_0. Fig. 2 proves clearly the reality of the sudden change which occurred on or about the 26th August. Dashed lines indicate $\left(\frac{dP}{dn}\right)_{t1}$ and $\left(\frac{dP}{dn}\right)_{t2}$ respectively. (There is certainly a wide gap between the observed and the theoratical values.) Weights of singular points are marked on each of the figures by ●● ○○ (in decreasing order). Parameters of major importance for the computations are summarized in the first columns of Table 6 ($i, e\,\varkappa$ and $\varphi_0\,\mu_0$, latters representing the reference latitude). Approximate orbital elements have been found in [12] and have been corrected according to the method mentioned above.

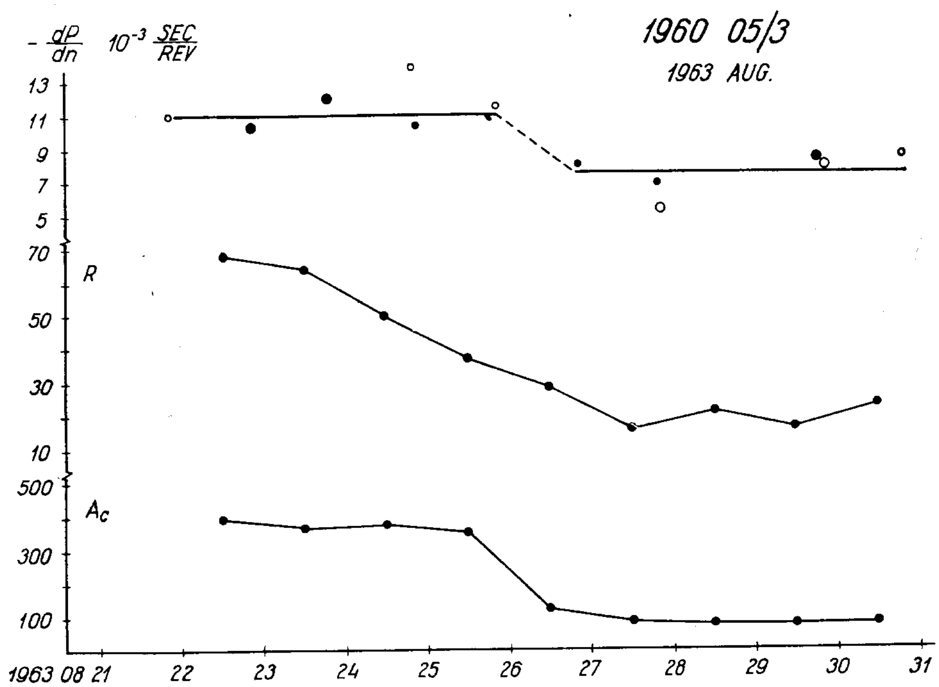

1960 05/3
1963 AUG.
$-\frac{dP}{dn}$ 10^{-3} $\frac{SEC}{REV}$
13
11
9
7
5
70
R
50
30
10
500
A_c
300
100
1963 08 21
22
23
24
25
26
27
28
29
30
31

Fig. 3.

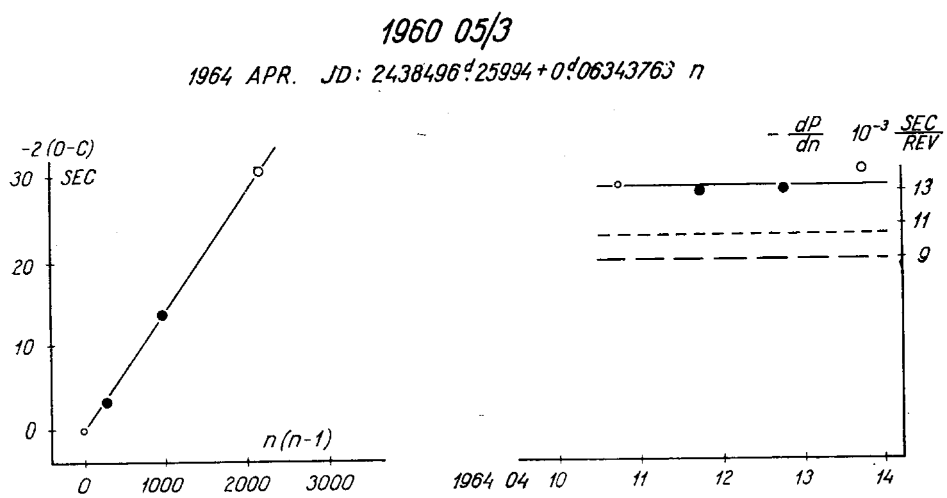

1960 05/3
1964 APR. JD: 2438496^d.25994 + 0^d.06343763 n
-2 (O-C)
SEC
30
20
10
0
n(n-1)
0
1000
2000
3000
$-\frac{dP}{dn}$ 10^{-3} $\frac{SEC}{REV}$
13
11
9
1964 04 10
11
12
13
14

Fig. 4.

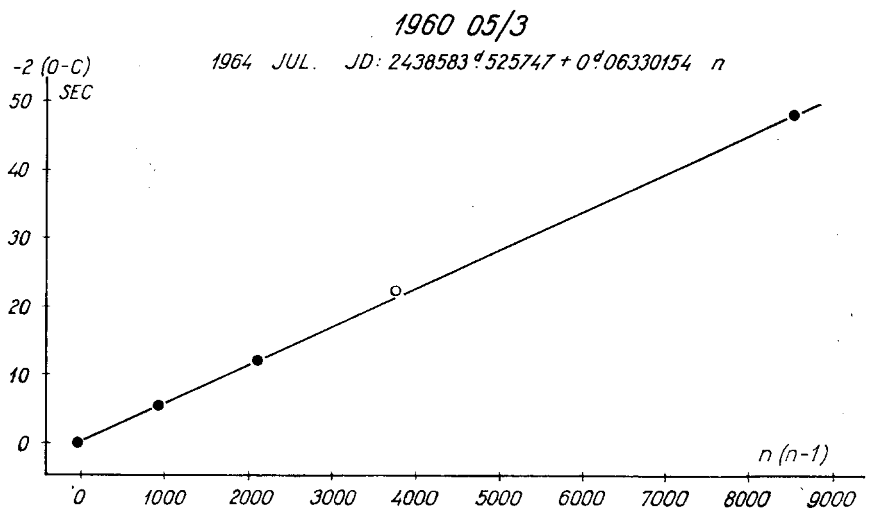

1960 05/3
1964 JUL. JD: 2438583^d525747 + 0^d06330154 n
-2 (O-C) SEC
50
40
30
20
10
0
n (n-1)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000

Fig. 5.

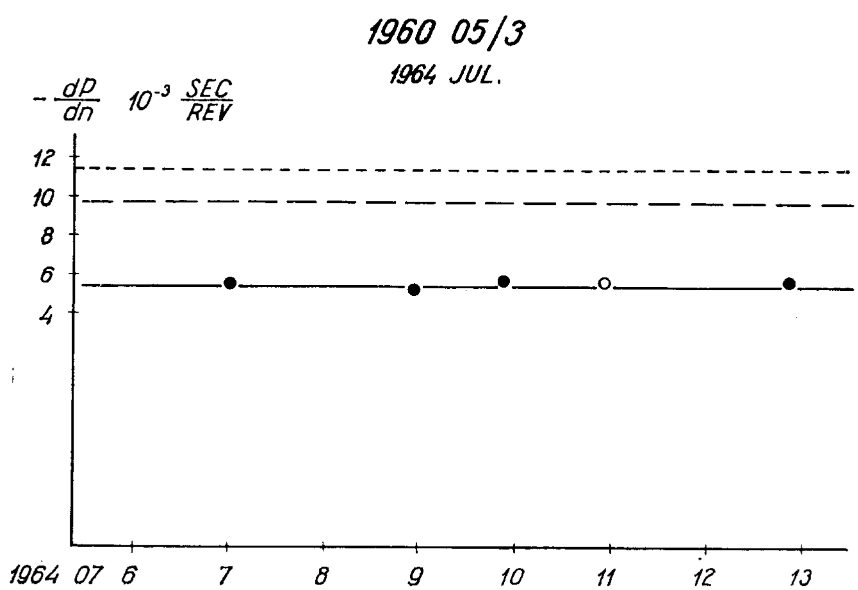

1960 05/3
1964 JUL.
- dP/dn 10^-3 SEC/REV
12
10
8
6
4
1964 07 6
7
8
9
10
11
12
13

Fig. 6.

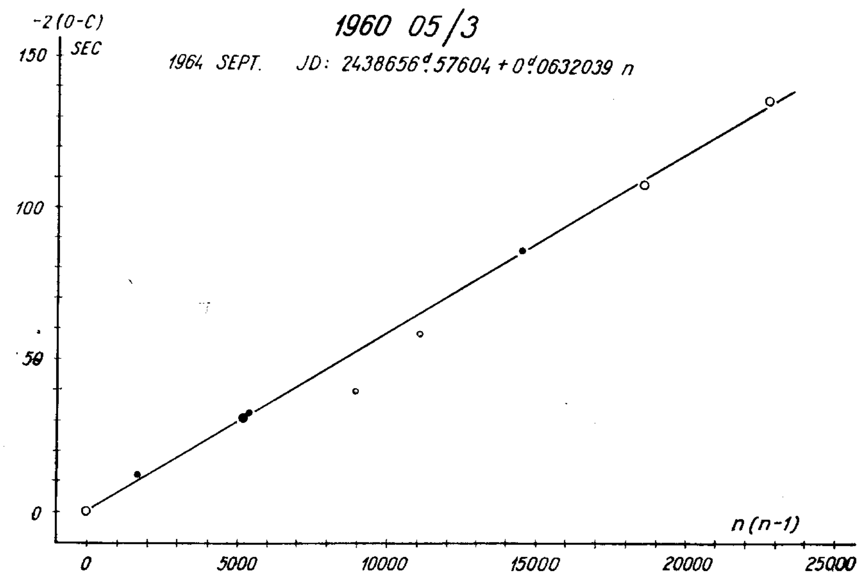
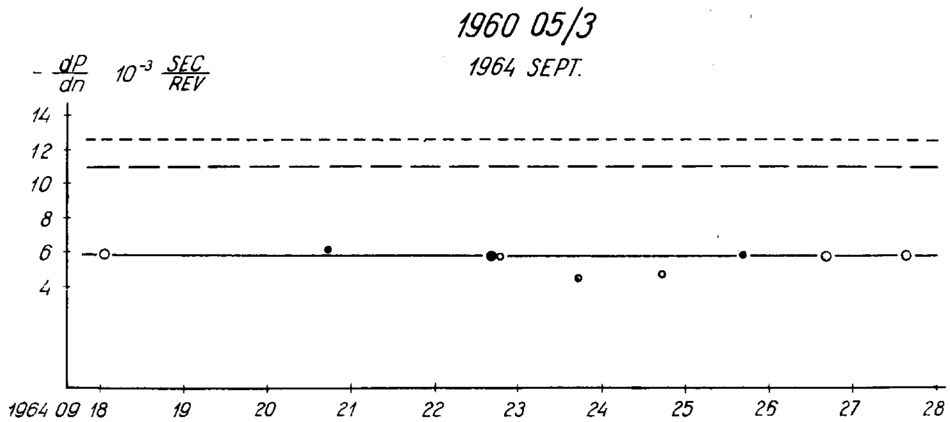


Fig. 7.



Fig, 8.

The last four columns give U_s (derived from $\bar{P}$), a, and the values of h_{ap} and h_{per} — being simple functions of a and e. The sidereal period of revolution can be computed from

$$U_s = P\left(1 + \frac{p}{v}\right)$$

[6], where

$$p = 9.97\left(\frac{R}{a}\right)^{3.5} \frac{\cos i}{\sqrt{1 - e^2}} \approx 3.5°/\text{day}$$

is the daily precession of the orbital plane,

$$v = \frac{|\mu_{n2} - \mu_{n1}|}{t_{n2} - t_{n1}} \approx 3.9°/\text{min}$$

is the average angular velocity easily measurable on the observed path elements. The sidereal period proved to be longer than the nodal one by 3—4 sec.

Consequently satellite accelerations as derived refer practically to a perigee altitude of 255 km.

CONCLUSIONS

The Interobs program sets as an aim the investigation of sudden changes in the upper atmosphere as revealed by satellite drag. Studying the observational data on 1960 ε3 (1960 05/3) we demonstrated the possibility of regarding $\frac{dP}{dn}$ as constant for time intervals of 8—10 days (at least during and about the minimum of solar activity) and to derive its mean value with no more than 20% external relative error. Consequently the density of the atmosphere near the perigee altitude of the satellite proved to be constant during most of the time intervals under consideration. Nevertheless we did not intend to determine it numerically for the following reasons: we did not succeed to derive the argument of perigee (the observations covering only a small fraction of the whole orbit) and also the effective cross-sectional area of the satellite is unknown. As the possibility of a slow precessional motion of the axis of rotation of the satellite (with a period of rotation of the order of weeks!) cannot be precluded [13], absolute measurements of air density are hampered by the corresponding variations of the cross-sectional area of the satellite at least if only one satellite is under observation.

Sudden variations, like the one shown on Fig. 2 and 3, however, cannot be regarded as anything else but as the consequences of sudden changes in the density of the upper atmosphere. Hence according to the changes of drag acting on 1960 ε3, air density near its perigee altitude decreased by 31% on the 26th August 1963. The change exceeds six times the standard error of $\frac{dP}{dn}$ for the preceding time interval.

It is natural to try to find a connection between this density decrease and variations in the actual solar activity, because a strong correlation between certain solar phenomena and the upper atmosphere is well known [14]. Exactly on the day mentioned a larger sunspot group turned away to the farside of the Sun, and, as a result, the Wolf (sunspot) number (R) dropped to about one half of its previous level. At the same time according to Catanian measurements the spot area (A) decreased to less than one third [15]. The coincidence of the solar and upper atmospheric events is interesting (see Fig. 3). Significant flares or distinctive events have not been observed during the time interval under consideration.

In 1964 solar activity had reached its absolute minimum, the average sunspot number being only 4 in April and July, 5 in September [15]. There were no visible spots on the Sun during 7—12 July and 16—28 Sept; average sunspot numbers being as low as 7—8—9 in April. It is by no means surprizing that the acceleration $\frac{dP}{dn}$ proved to be constant during these time intervals, and its absolute value in April 1964 was considerably larger than in July or in September. The correlation "more intensive solar activity — more intensive drag" approximately holds true, suggesting that with decrease in solar activity the upper atmosphere becomes cooler and shrinks closer to the earth so that at a given geometric altitude the density becomes less.

The comparison of $\frac{dP}{dn}$ values obtained for different time intervals demands, however, a lot of precaution because of the shift of the perigee and changes of the effective cross-sectional area as mentioned above. The general tendency — i.e. that acceleration contrary to all expectations is a *decreasing* function of time — has been proved by the convex shape (as seen from below) of the $\bar{P}(t)$ plot independently. From the graphically derived slopes of the curve at different points we find $\left|\frac{dP}{dn}\right|^*$ decreasing as well (see Table 5); even the arithmetical means of the $\left(\frac{dP}{dn}\right)^*$ and $\frac{dP}{dn}$ values derived during the four time intervals under consideration are practically equal (-0.00839 and -0.00861 s/rev respectively). Accordingly our results suggest that during 1963/64 air density around the altitude 250—260 km decreased — parallel with solar activity — to such an extent that the drag encountered by satellite 1960 ε3 did not show a rising but a falling tendency considerably prolonging the lifetime of the satellite. It would be an interesting task to determine whether similar effects occurred in the motion of other satellites at the same time; unfortunately the Interobs material available did not allow such an investigation.

Studying all observations of 1960 ε3 in 1963/64 it could be ascertained that in time intervals when at least 3 transits have been well observed (simultaneously from two stations), the rate of change of the period of revolution, if constant, can be determined with an error not exceeding 10—15%. Supposing that sudden variations occur in air density which has an effect on $\frac{dP}{dn}$ larger than this value, the date of change can be computed with an accuracy of possibly less than one day (in a fortunate case) or at least 1—2 days. Such time resolution is of interest when investigating the time delay of the influence of certain solar phenomena upon the upper atmosphere. It seems useful to extend the Interobs program in order to obtain material rich enough to set up actual correlations.

Konkoly Observatory, Budapest, September 1965.

APPENDIX

PROGRAM FOR ELLIOTT 803 COMPUTER TO CALCULATE THE SATELLITE'S POSITION IN SPACE FROM SIMULTANEOUS OBSERVATIONS

This program forms only the first part of a larger one to evaluate orbital elements directly from simultaneous observations (see [9]). The output of some data in excess is needed for the second part.

Input data

No. 1. number of tracking stations (less than 40);
No. 2. coordinates of tracking stations in a rectangular and in a geographic system:

$$(x^*, y^*, z^*, \lambda, \sin\varphi, \cos\varphi)_i \qquad (i = 1, 2, \ldots) \text{ (see [9])}$$

No. 3. number of days when observations have been performed (less than 200);
No. 4. hours, minutes and seconds of Greenwich sidereal time at 0^h GMT of days when observations have been performed;
No. 5. A_1 and A_2 (Parameters in

$$\alpha_m = \alpha_n + A_1 + A_2 \sin\alpha_n \operatorname{tg}\delta_n$$

$$\delta_m = \delta_n + A_2 \cos\alpha_n$$

where m is the year of observation,
n is the epoch of the catalogue or atlas);
No. 6. year, month, day and hour (integer numbers) of the first transit;
No. 7. identification number of the satellite (integer of maximum 5 figures);
No. 8. observing data: weight, time (h, m, s), identification number of the coordinate system (1 means equatorial, 2 means horizontal), right ascension or azimuth (h, m, s or °, ', ") declination or elevation (°, ') serial number of the tracking station according to No. 2, and serial number of the day according to No. 4. N.B. Maximum number of simultaneity groups (points in space) on each transit is 100. More than one observation from the same tracking station cannot be used in one simultaneity group.

Arriving to the next transit write all input data (No. 8) of the first observation, then 38(40(and the time of the new transit as in No. 6. In the case of a new satellite add 39(and the identification number of the satellite as in No. 7. After the very last observation add 12 zeros and 38(34( ).

Output

Every transit is printed out as a separate group. The first line gives: 1. the identification number of the satellite, 2. year, month, day and hour of the transit.
Line No. 2: time (in degrees) and θ (in degrees, see [9])of the first real observation;
Line No. 3: $\Sigma x_s^2, \Sigma y_s^2, \Sigma z_s^2$ for the given transit. Spatial coordinates are tabulated below as follows:

$$x_{si}, y_{si}, z_{si}, R_{si}, n_i \qquad (i = 1, 2, \ldots)$$

where n_i is the number of all possible pairs in the simultaneity group when determining the satellite's position in space. The sign 5(at the end of every transit belongs to **Part II.** of the program.

After the last transit all observations neglected during the calculation are printed out using the following code:
Starting from horizontal coordinates

111111 indicates $\sin \delta > 0.999$
222222 indicates $|\sin t| > 0.99996$
333333 indicates $|\sin a| > 0.99996$ and

starting from equatorial coordinates

444444 indicates $\delta > 87°.5$
555555 indicates $|\sin a| > 0.99996.$

Numbers after these six-figure symbols give the serial number of the observation under consideration.

Further, whilst determining a position in space, those pairs of observations are not used where

$$|\text{tg } a_K - \text{tg } a_l| < 0.01$$

these are indicated by the symbol 666666 and k and l are printed out afterwards.

```
SETS LIHJUVKQWCD(2002)E(4)
SETV P(240)A(205)B(23)S(160)G(10)
   N(11)Z(400)
SETF TRIG ARCTAN SQRT
SETR 40

1) READ L
VARY I=1 : 1 : L
VARY H=0 : 40 : 6
READ P(I+H)
REPEAT H
REPEAT I
READ L
VARY I=1 : 1 : L
READ A(I)
READ A(I+1)
READ A(I+2)
A(I+2)=.01666666 * A(I+2)
A(I+1)=A(I+2)+A(I+1)
A(I+1)=.01666666 * A(I+1)
A(I)=A(I)+A(I+1)
A(I)=15 * A(I)
REPEAT I
READ G9
READ G10
N1=0
N4=0
N8=0
VARY I=1 : 1 : 4
READ EI
REPEAT I
2) READ V
C=1001
H=0
21) D2001=0
J=1
U=0
22) READ S(J)
VARY I=1 : 1 : 9
READ B(I)
REPEAT I
READ Q
READ W
JUMP IF J=1TO20
JUMP UNLESS B3=B13TO3
JUMP UNLESS B2=B12TO3
JUMP UNLESS B1=B11TO3
JUMP TO5
20) B11=B1
B12=B2
B13=B3
B3=.01666666 * B3
B2=B2+B3
B2=.01666666 * B2
B1=B1+B2
B20=15*B1
B1=1.00273791 * B20
B14=B1+A(W)
23) JUMP UNLESS B14>360TO4
B14=B14-360
JUMP TO23
4) B1=B14/180
B15=SIN B1
B16=COS B1
5) B1=P(Q) * B16
B2=P(Q+40) * B15
S(J+4)=B1-B2
B1=P(Q) * B15
B2=P(Q+40) * B16
S(J+5)=B1+B2
S(J+6)=P(Q+80)
JUMP IF B4=1TO16
JUMP IF B8>60TO10
JUMP IF B8>30TO9
JUMP IF B8>20TO8
JUMP IF B8>15TO7
```

```
JUMP IF B8>12TO6
B9=B9-5
JUMP TO10
6) B9=B9-4
JUMP TO10
7) B9=B9-3
JUMP TO10
8) B9=B9-2
JUMP TO10
9) B9=B9-1
10) B7=.01666666 * B7
B6=B7+B6
B6=.01666666 * B6
B5=B5+B6
B9=.01666666 * B9
B8=B9+B8
B5=B5/180
B6=COS B5
B5=SIN B5
B8=B8/180
B9=COS B8
B8=SIN B8
B1=P(Q+160) * B8
B2=P(Q+200) * B9
B2=B2*B6
B1=B1+B2
JUMP UNLESS B1>.999TO24
D(C)=111111
C=C+1
U=U+1
D(C)=U
C=C+1
JUMP IF C>1997TO34
JUMP TO22
24) B2=B1*B1
B2=1-B2
B2=SQRT B2
S(J+3)=B1/B2
B3=-B9*B5
B2=B3/B2
B3=MOD B2
JUMP UNLESS B3>.99996TO25
D(C)=222222
C=C+1
U=U+1
D(C)=U
C=C+1
JUMP IF C>1997TO34
JUMP TO22
25) B3=P(Q+160) * B1
B3=B8-B3
B7=B2*B2
B7=1-B7
B7=SQRT B7
B7=B2/B7
B7=ARCTAN B7
B7=180 * B7
JUMP IF B2>0TO11
JUMP IF B3>0TO12
B7=-180-B7
JUMP TO12
11) JUMP IF B3>0TO12
B7=180-B7
12) B1=B14-B7
B1=B1-P(Q+120)
26) JUMP IF B1<0TO14
13) JUMP IF B1<360TO15
B1=B1-360
JUMP TO13
14) B1=B1+360
JUMP TO26
15) B1=B1/180
B2=SIN B1
B3=MOD B2
JUMP UNLESS B3>.99996TO27
D(C)=333333
C=C+1
U=U+1
D(C)=U
C=C+1
JUMP IF C>1997TO34
JUMP TO22
27) B1=COS B1
S(J+1)=B2/B1
S(J+7)=S(J+4)*(J+1)
S(J+2)=1/B1
JUMP TO19
16) B7=.01666666*B7
B6=B6+B7
B6=.01666666*B6
B5=B5+B6
B5=15*B5
B9=.01666666*B9
B8=B8+B9
B7=B5/180
B6=COS B7
B7=SIN B7
B7=G10*B7
B6=G10*B6
B9=B8/180
B9=TAN B9
B7=B7*B9
B7=B7+G9
B5=B5+B7
B8=B8+B6
JUMP IF B8<90TO17
B8=180-B8
B5=B5+180
17) JUMP UNLESS B8>87.5TO28
D(C)=444444
C=C+1
U=U+1
D(C)=U
C=C+1
JUMP IF C>1997TO34
JUMP TO 22
28) JUMP UNLESS B5<0TO29
B5=360+B5
JUMP TO28
29) JUMP IF B5<360TO18
B5=B5-360
JUMP TO29
18) B8=B8/180
S(J+3)=TAN B8
B5=B5/180
B2=SIN B5
```

```
B8=MOD B2
JUMP UNLESS B8>.99996TO30
D(C)=555555
C=C+1
U=U+1
D(C)=U
C=C+1
JUMP IF C>1997TO34
JUMP TO 22
30) B5=COS B5
S(J+1)=B2/B5
S(J+7)=S(J+4)*S(J+1)
S(J+2)=1/B5
19) U=U+1
D2001=D2001+1
JUMP UNLESS D2001=1TO37
B22=B20
B23=B14
37) J=J+8
JUMP TO22
3) I=0
N2=0
N3=0
N5=0
N6=0
N7=0
K=1
L=K+8
31) G7=S(K)*S(L)
N5=N5+G7
G8=S(K)+S(L)
N6=N6+G8
G1=S(L+5)-S(K+5)
G1=G1-S(L+7)
G1=G1+S(K+7)
G2=S(K+1)-S(L+1)
G6=MOD G2
JUMP UNLESS G6<.1TO32
D(C)=666666
C=C+1
D(C)=K
C=C+1
D(C)=L
C=C+1
JUMP UNLESS C>1997TO35
LINE
D2002=C-1001
VARY C=1001 : 1 : D2002
PRINT D(C)
LINE
REPEAT C
C=1001
JUMP TO35
32) G1=G1/G2
G6=G1*G7
N7=N7+G6
G2=G1-S(K+4)
G3=G2*S(K+1)
G3=S(K+5)+G3
G5=G3*G7
N2=N2+G5
G2=G2*S(K+3)
G2=G2*S(K+2)
G2=G2+S(K+6)
G4=S(K)*G2
N3=N3+G4
G1=G1-S(L+4)
G1=G1*S(L+3)
G1=G1*S(L+2)
G1=S(L+6)+G1
G1=G1*S(L)
N3=N3+G1
I=I+1
35) L=L+8
JUMP IF L<JTO31
33) K=K+8
L=K+8
JUMP IF L<JTO31
D(H)=I
Z(4H+1)=N7/N5
Z(4H+2)=N2/N5
Z(4H+3)=N3/N6
B17=Z(4H+1)*Z(4H+1)
N1=N1+B17
B18=Z(4H+2)*Z(4H+2)
N4=N4+B18
B19=Z(4H+3)*Z(4H+3)
N8=N8+B19
B17=B17+B18
B17=B17+B19
Z(4H+4)=SQRT B17
H=H+1
S1=S J
J=1
JUMP TO20
38) LINE
PRINT V,5
SPACES 5
PRINT E1,4
PRINT E2,2
PRINT E3,2
PRINT E4,2
LINE
PRINT B22
PRINT B23
LINE
PRINT N1
PRINT N4
PRINT N8
LINE
VARY I=0 : 1 : H
VARY K=1 : 1 : 4
PRINT Z(4I+K)
REPEAT K
PRINT DI
LINE
REPEAT I
OUTPUT 21
OUTPUT 17
N1=0
N4=0
N8=0
D2001=0
H=0
JUMP TO22
34) C=C-1
```

```
LINES 3
CYCLE K=1001 : 1 : C
PRINT DK,6
LINE
REPEAT K
C=1001
JUMP TO22
39) READ V
JUMP TO22
40) VARY I=1 : 1 : 4
READ EI
REPEAT I
JUMP TO22
36) STOP
START 1
```

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