THE OPACITY AND THE INTERNAL STRUCTURE OF THE SUN

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I. Introduction

The purpose of this paper is to obtain the distributions of the temper a ture, density and pressure from the surface to the center of the sun on the one hand and to get the composition, i. e. the content of hydrogen, helium and heavy elements, of the sun and the relative importance of the proton-proton reaction and the carbon cycle to the contribution of the solar energy on the other hand.

There are many papers, such as those of *Eddington* [1], *Strömgren* [2], *Cowling* [3], *Chandrasekhar* [4], *Ledoux* [5], Canadian [6] and Soviet [7] workers, to investigate the stellar structure in general. Only a few articles, however, have been published to deal with the structure of the sun by means of numerical calculation. Since 1939 *Bethe* [8] and *Weizsäcker* [9] suggested independently that the carbon cycle provided the stellar energy, *Schwarzschild* [10] first computed the solar model with the carbon cycle as the source of the solar energy and obtained the composition of hydrogen, helium and heavy elements for the sun. He employed a simple approximate formula for the opacity law inside the sun.

Later in 1949, *Harrison* [11] investigated the problem again along the line of Schwarzschild's, but used a better opacity formula which followed generally the values obtained according to the temperature and the density distributions of her solar model. In 1950 *Epstein* [12] pointed out the importance* of the proton-proton reaction to the contribution of the solar energy.

Until 1953 the opacity inside the sun was considered to be wholly due to heavy elements. Later, it was realized that the contribution of the free-free absorptions of hydrogen and helium to the opacity could not be neglected, mainly due to the extremely low content of heavy elements. Then *Naur* [13], later *Epstein* and *Motz* [14] have taken the free-free absorptions of hydrogen and helium into consideration and computed the solar models.

Table 1 lists the essential results of the solar models of current authors. All the models except one of Naur's in Table 1 are the so-called Cowling model which has a convective core and a radiative envelope. Each model has a common assumption that the mean molecular weight is the same inside the sun. The validity of this assumption depends wholly upon the thoroughly mixing of the material between the core and the envelope, otherwise the mean molecular weight in the core will be higher than that in the envelope due to the more effective transformation of hydrogen into helium in the

* Epstein misused Mrs. Harrison's results and reached a conclusion which exaggerated the importance of the proton-proton reaction in his article, but his general conclusion is still true.

Table 1 Results of Current Solar Models

Author	Date	Opacity Law	X	Y	Z	$T_c \times 10^{-6}$	ϱ_c g/cm³	Energy Source	Core radius r/R
Schwarzschild	1946	$K_0 \varrho^{0.75} T^{-3.5}$ H, He not consid.	,47	,41	,12	19,8	111,6	C-cycle	,122
Harrison	1948	K (T, ϱ) H, He not consid.	,653	,279	,068	19,8	196,2	C-cycle	,091
Epstein and *Motz*	1953	K (T, ϱ) H, He not consid.	,998	0	,002	12,0	110	C-cycle & P—P reaction	,080*
Naur	1954	[1] $K_0' \varrho^{0.75} T^{-3.5}$ $K_0'' \varrho T^{-3.5}$	,74	,25	,0075	13,5	94	P—P reaction	,000
		[2] $K_0''' T^{-0.9}$ $K_0^{IV} T^{-3.8}$ $(H + He)_{ff}$ consid.	,76	,23	,0075	13,8	85,7	P—P reaction	,050
Epstein & *Motz*	1954	K (T, ϱ) $(H)_{ff}$ consid.	,931	0,67	,002	12,8	100	C-cycle & P—P reaction	,080

* In this article the values of $(n+1)$ discontinue at some point. At the interface of the core and envelope the value of $(n+1)$ is equal to 2,75 instead of the correct value 2,50 (Ap. J. *117*, 311, 1953)

core. Whether is there thoroughly mixing of the material inside the sun is not settled. However, *Greenstein* and others [15] pointed out that the appearance of beryllium and lithium in the solar atmosphere seemed in favor of non-mixing.

We have investigated this problem in general along the lines of the previous works except that we assumed the mean molecular weight in the core different from that in the envelope. Since 1955 we have computed a series of solar models with both the proton-proton reaction and carbon cycle as the source of the solar energy. The latest data of nuclear reactions given by *Fowler* [16, 20] are employed. In the computation of opacities accordingly to the temperature and the density distributions of our assumed models, we found out that not only the free-free absorptions, but the bound-free absorptions of hydrogen and helium should be taken into account as well in the outer layers of the sun, where the temperature is lower than 2×10^6°K. This is partly due to the extremely low content of heavy elements, 0,1 per cent in our model and partly due to the fact that at low temperatures the peak of energy distribution of the Planckian function shifts to the longer wave length and thus the photo-ionization of K and L electrons of hydrogen and helium plays a leading role in contributing to opacity, as pointed out by *Keller* and *Meyerott* [17].

The values of opacity are computed according to the density distribution of assumed models at 14 points with assigned temperatures. The solar model with a hydrogen content of 82,5 per cent gives the value of energy generation very close to the observed one, apart from that the values of density distribution of the resultant model is much higher than that of assumed mode at the region near the surface. This will produce the same order of discrepancy between the values of opacity of the resultant and assumed models, and, so we have to start new models. From successive computations with different density distributions we obtain an empirical relation between the density distributions of the assumed and the resultant models. It can be stated generally as follows: the higher is the values of the density distribution of the assumed model, the lower is the values of the resultant model. With this emperical relation in mind, it is easier for us to assume a density distribution more close to the one of the resultant model.

II. *The Opacity of the Sun*

The opacity $\varkappa$ is defined by

$$\frac{1}{\varkappa}=\frac{\int\limits_0^\infty \frac{1}{k_\nu}\frac{1}{(1-e^{-h\nu/kT})}\frac{dB_\nu}{dT}\,d\nu}{\int\limits_0^\infty \frac{dB_\nu}{dT}\,d\nu}, \tag{1}$$

where B_ν is the Planckian function, k_ν is the absorption cœfficient per gram at frequency ν and the other quantities have their usual meanings. According to *Strömgren*'s computation [18], $\varkappa$ can be written as

$$\varkappa=3{,}90\times 10^{25}\cdot\frac{\Gamma}{\Gamma_R}\,\varrho\,T^{-3,5}(1+X)\,\frac{1}{\tau}, \tag{2}$$

Γ, Γ_R are, the composition, respectively, for the adopted composition and for the Russell mixture,

$$\Gamma=\Sigma\,\frac{c_z Z^2}{A}, \qquad \Gamma_R=6; \tag{3}$$

c_z, the percentage in the composition of certain element whose atomic number and weight are, respectively, Z and A. τ in formula (2) is the guillotine factor and calculated by the following formulae:

$$\chi_n=\frac{2\,\pi^2 e^4\, mZ^2}{n^2 h^2} \tag{4}$$

$$D_n=D_{j,e}\,\frac{2}{n}\,\frac{\chi_n}{kT}\,\frac{e^{\frac{\chi_n}{kT}}}{1+exp\left(\frac{\chi_n}{kT}-\frac{\psi}{kT}\right)} \tag{5}$$

$$\tau(\chi_i,\chi_{i+1})=\frac{\Phi(\chi_i)-\Phi(\chi_{i+1})}{D(\chi_i,\chi_{i-1})}, \qquad \chi_i=\frac{h\nu_i}{kT}, \tag{6}$$

$$\tau=\Sigma\,\tau(\chi_i,\chi_{i+1}) \tag{7}$$

On applying the above formulae to obtain the guillotine factors, we compute them according to the following two principles: (1) in the computation of the absorption coefficients we, taking into account the effect of the excluded volume, have included only those terms having quantum number $n \leq 4$; (2) with regard to the depression of the continuum we have excluded the terms having quantum number, $n \geq 2$, for the bound-free absorption of hydrogen and helium when the density is high. This latter limitation is not important for the solar model because the bound-free absorptions of hydrogen and helium become negligible due to the increasingly high temperature when the density is high.

The composition of heavy elements we have adopted is 40 per cent Russell mixture and 60 per cent oxygen. As the values of composite factor for heavy elements, hydrogen and helium are, respectively, 4,8, 1 and 1, we have the resultant D_n at ν_n as,

$$D_n = \frac{4{,}8\, c_z D_n(Z) + C_H\, D_n(H) + C_{He}\, D_n(He)}{\Gamma}, \tag{8}$$

where
$$\Gamma = 4{,}8\, c_z + (C_H + C_{He}) \times 1$$

The values of guillotine factor so computed at 14 assigned temperatures and at proper values of ψ/kT for including the temperature-density points of our model are given in Table 2. The resultant opacities consisting of the atomic

Table 2 Guillotine Factors $\text{Log}_{10}\,\tau$ and Densities $\text{Log}_{10}\,\varrho$ for the Equilibrium model of the Sun $X = 0{,}820$, $Y = 0{,}179$, $Z = 0{,}001$ (60% oxygen and 40% Russell mixture)

$T \times 10^{-6}$	ψ/kT	$\log_{10}\tau$	ψ/KT	$\log_{10}\tau$	$\log_{10}\varrho$ assumed	$\log_{10}\varrho$ resultant
0,300	8	1,123	9	1,104	4,430	$\bar{4}$,423
0,625	6	1,670	7	1,614	3,700	$\bar{3}$,715
0,714	6	1,718	7	1,615	$\bar{3}$,930	$\bar{3}$,943
0,833	6	1,751	7	1,692	$\bar{2}$,181	2,193
1,000	5	1,842	6	1,775	$\bar{2}$,452	$\bar{2}$,481
2,000	4	1,889	5	1,763	1,468	$\bar{1}$,482
3,000	3	2,110	4	2,014	0,104	0,096
5,000	2	2,242	3	2,198	0,879	0,885
6,000	2	2,250	3	2,215	1,130	1,144
7,500	1	2,276	2	2,259	1,410	1,434
9,000	1	2,276	2	2,263	1,610	1,637
10,000	1	2,277	2	2,264	1,709	1,739
12,000	1	2,280	2	2,264	1,850	1,885

opacity and electron scattering are obtained by means of the tabulated values given by *Morse* [19]. The values of the resultant opacity are represented by the following four equations:

$$\left.\begin{array}{lll} \log \varkappa_1 = -1{,}0533 \log T + 7{,}0309 & & \log T \leq 5{,}796 \\ \log \varkappa_2 = -0{,}5588 \log T + 4{,}1649 & & 5{,}796 \leq \log T \leq 6{,}000 \\ \log \varkappa_3 = -0{,}2160 \log T + 2{,}1077 & & 6{,}000 \leq \log T \leq 6{,}301 \\ \log \varkappa_4 = -0{,}8213 \log T + 5{,}9223 & & 6{,}301 \leq \log T \end{array}\right\} \tag{9}$$

In the current works there are three papers in which the free-free absorptions of hydrogen and helium have taken into consideration. The correctness of their formulae to combine the opacity of heavy elements and of hydrogen and helium is worthy to be discussed. *Naur*'s opacity formula [13] is

$$\begin{aligned} \varkappa = 3{,}45 \times 10^{25} (1 + X)\, Z \varrho T^{-3,5} \frac{g}{t} + \\ + 2{,}7 \times 10^{22} (1 + X)(X + Y)\, \varrho T^{-3,5} + 0{,}3 (1 + X), \end{aligned} \tag{10}$$

in which the resultant opacity is equal to the sum of the opacity of heavy elements, that of free-free absorptions of hydrogen and helium and 1,5 times the scattering of the electrons. Next, the *Abell*'s opacity formula [20] is

$$\begin{aligned} \varkappa &= \frac{1}{\tau} k \varrho T^{-3,5} + 0{,}3 (1 + X) \\ k &= 3{,}9 \times 10^{25} (1 + X)(1 - X - Y) + 4{,}10 \times 10^{22} (1 + X)(1 + Y), \end{aligned} \tag{11}$$

in which the resultant opacity is equal to the sum of the opacity of the heavy elements, 1,5 times that of the free-free absorption of hydrogen and 1,5 times the scattering of electrons. Actually, the resultant opacity, as given by equation (1), is equal to the weighted harmonic mean of absorption coefficients of various constituents and electron scattering, and naturally, will not be equal to those given by their formulae. Therefore, both Naur and Abell did not employ their formulae just given, in actual computation of opacity, but used the more general ones as indicated in our Table 1. Furthermore, *Strömgren*'s empirical formula (2) for the resultant opacity (i. e. the larger value plus 1,5 times the smaller one) consisting both of the absorptions of heavy elements and of electron scattering is good approximately for the case of early B-stars. It is questionable whether it can be applied as well for the G-type star such as the sun and even more doubtful when it is applied to the case for the free-free absorptions of hydrogen and helium.

Epstein and *Motz* [14] combine the free-free absorption of hydrogen with that of heavy elements to get the guillotine factor t. Their opacity formula is given by

$$\varkappa = 3{,}9 \times 10^{25} \varrho\, T^{-3,5} (1 + X)(1 - X - Y) \frac{1}{t}. \tag{12}$$

As to check our computation we have computed the opacity according to their composition ($X = 0{,}931$, $Y = 0{,}067$ and $Z = 0{,}002$) and the temperature and density distributions of their model. With equations (2) and (3) of the present paper, we obtain

$$\varkappa = 3{,}9 \times 10^{25} \frac{1{,}0076}{6} \varrho\, T^{-3,5} (1 + X) \frac{1}{\tau}. \tag{13}$$

Since $(1 - X - Y) = 0{,}002$, the relation between t and τ is

$$\mathrm{Log}_{10} \tau = 1{,}924 + \mathrm{Log}_{10} t. \tag{14}$$

The values of $\log_{10} \tau$ obtained from [14] and those of us are tabulated in table 3 in which for the purpose of comparison both values, besides consisting of the absorptions of the heavy elements and free-free absorptions of hydrogen and helium, the one with (with the bracket (b)) and the other without the bound-free absorptions of hydrogen and helium, are included. From table 3 one can see, there are quite differences between the two sets of the values of $\mathrm{Log}_{10}\,\tau$.

Table 3. Comparison of the two sets of values of $\mathrm{Log}_{10}\,\tau$, for $X = 0,931$, $Y = 0,067$, $Z = 0,002$

$T \times 10^{-6}$	ψ/kT	Kung	E & M	ψ/kT	Kung	E & M
0,625	6	1,774 1,621 (b)*	1,898			
0,714	6	1,808 1,667 (b)	1,890			
0,833	5	1,994 1,806 (b)	1,712	6	1,815 1,691 (b)	1,791
1,000	5	1,905 1,793 (b)	1,609	6	1,815 1,709 (b)	1,686
3,000	3	2,207 2,019 (b)	2,288	4	1,890 1,884 (b)	2,142
5,000	2	2,197 2,197 (b)	2,556	3	2,122 2,122 (b)	2,380
6,000				2	2,122 2,122 (b)	2,690

* The values with bracket (b) contain also the contribution of the bound-free absorptions of hydrogen and helium.

Furthermore, some values of $\varkappa$ (i. e. $\varkappa$ in Epstein and Motz's paper) obtained by equation (6) of their paper by employing values of t in their Table 2 corresponding to the temperature and density distributions of their model have appreciable differences with those obtained by equation (5) of their paper. (The second formula of equation (5) has a printing error. The constant should be $+0,588$, instead of $-0,588$). These two sets of values should have little difference. They are shown in Table 4.

Table 4. Values of $\mathrm{Log}_{10}\,\varkappa$

$T \times 10^{-6}$	0,714	0,833	1,0	3,0	5,0	6,0	7,5	9,0	10,0
From (6)	0,794	0,976	1,171	0,380	0,144	$\bar{1},947$	0,208	$\bar{1},825$	$\bar{1},710$
From (9)	0,707	0,641	0,562	0,467	0,153	0,041	$\bar{1},903$	$\bar{1},792$	$\bar{1},727$
Difference	+,087	+,335	+,609	—,087	—,009	—,094	+,239	+,033	—,017

According to equations (5) and (6) of this paper, it is evident that the values of $\mathrm{Log}_{10}\,\tau$ at the same temperature should decrease as the values of ψ increase. This is clearly illustrated in *Harrison*'s Tables (11). It is rather perplexing to read some values of $\mathrm{Log}_{10}\,t$ in Table 2 of Epstein and Motz's paper that at $T = 0{,}625$, $0{,}833$ and $1\mathrm{x}10^6$ degrees, the values of $\mathrm{Log}_{10}\,t$ at $\psi/kT = 5, 6$, and ∞ increase with the values of ψ/kT, instead of decreasing.

III. *The Structure of the Sun*

The differential equations used in this investigation are based upon the following assumptions :

(1) The sun may have a convective core or an isothermal core or no core at all. If a convective core exists, the material inside the core, like an ideal gas, has a value of $\gamma = c_p/c_v = 5/3$. If an isothermal core exists, the material inside the core has exhausted its hydrogen and so generates no energy. The material outside the core is in the state of radiative equilibrium.

(2) The mean molecular weight of the core must be larger than, or at least equal to that of the envelope.

(3) Radiation pressure can be neglected.

(4) The thermal nuclear reactions operate both in the convective core and in the radiative envelope.

The assumption (4) is introduced because the proton-proton reaction generates energy rather effectively under lower temperature range ($10^{7\circ}$—$1{,}3 \cdot 10^{7\circ}$). Since the rate of energy generation depends on the content of hydrogen and nitrogen, we must assume before-hand the content of hydrogen and heavy elements so as to carry out the numerical integration in the envelope. Thus the problem is solved by successive steps point by point.

The four differential equations are :

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\varrho\,, \tag{15}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \varrho\,, \tag{16}$$

$$\frac{dL(r)}{dr} = 4\pi r^2 \varrho\,(\epsilon_p + \epsilon_c)\,, \tag{17}$$

$$\frac{d(T^4)}{dr} = -\frac{3}{ac}\varkappa\varrho\,\frac{L(r)}{4\pi r^2}\,,\ \text{radiative equilibrium} \tag{18a}$$

$$\frac{1}{P}\frac{dP}{dr} = \gamma\,\frac{1}{\varrho}\frac{d\varrho}{dr}\,,\ \text{convective equilibrium} \tag{18a}$$

Where ϵ_p, ϵ_c are, the energy generated per gram per second, respectively, by the proton-proton reaction and carbon cycle. The other quantities have their usual meaning (4). As usual we employ the non-dimensional quantities as follows :

$$P = p\,\frac{GM^2}{4\pi R}\,,\quad T = t\,\frac{\mu H}{k}\,\frac{GM}{R}\,,\quad M(r) = qM\,,\quad r = xR,\quad L(r) = yL \tag{19}$$

The energy generation equations [16] [20] can then be put in the following forms :

$$\epsilon_c = 4{,}05 \times 10^{-23} \chi_{CN} (1 - X - Y) X p t^{19,3} \left(\frac{\mu H}{10^6 k} \frac{GM}{R} \right)^{20,3} \frac{M}{4 \pi R^3} = \epsilon_c^{\circ} p t^{19,3}, \qquad (20a)$$

$$\epsilon p = 5{,}42 \times 10^{-6} X^2 \frac{M}{4 \pi R^3} \left(\frac{\mu H}{10^6 k} \frac{GM}{R} \right)^{4,1} p t^{3,1} = \epsilon_p^{\circ} p t^{3,1} \qquad (20b)$$

Where χ_{CN} is the portion of carbon plus nitrogen content in the heavy elements, we take $\chi_{CN} = 0{,}2$.

In the process of numerical integration when $\log y = -\infty$, $\Delta \log t = 0$, it means that the energy generated outside the sphere of the radius in question attains the total energy of the sun. If the solution of the isothermal core does not exist, it infers that our assumed model is wrong. Quite a few models we assumed are such case of over production of energy.

When the value of $d(\log p)/d(\log t)$ decreases and is equal to 2,5 in the integration, it speaks that the boundary of the convective core is reached. With the values of t, p and q at the interface we can calculate the ratio of the values of non-dimensional quantities U and V by usual means. Then the Emden variable ξ_f corresponding to the radius at the interface by finding in Emden table the ratio of U and V equal to that just found. The relations between the Emden variables Θ, ξ and our previous ones are :

$$\left.\begin{aligned} T &= T_0 \Theta = \frac{T_f}{\Theta_f} \Theta = \frac{t_f \mu_e H}{\Theta_f k} \frac{GM}{R} \Theta, \\ \varrho &= \varrho_0 \Theta^{1,5} = \frac{\varrho_f}{\Theta_f^{1,5}} \Theta^{1,5} = \frac{p_f \omega}{\Theta_f^{1,5} t_f} \frac{M}{4 \pi R^3} \Theta^{1,5}, \\ r &= x R = \xi \frac{x_f}{\xi_f} R ; \end{aligned}\right\} \qquad (21)$$

where subindex f indicates the value at interface and ω is the ratio of the mean molecular weight in the core to that in the envelope. The total energy generated inside the core is

$$\begin{aligned} y_c(r) = {} & \frac{X_c}{X_e} \epsilon_c^{\circ} \left(\frac{t_f}{\Theta_f} \right)^{20,3} \frac{M}{L} \left(\frac{p_f \omega}{\Theta^{1,5} t_f} \right)^2 \left(\frac{x_f}{\xi_f} \right)^3 \int_0^{\xi_f} \Theta^{23,3} \xi^2 \, d\xi + \\ & + \frac{X_c^2}{X_e^2} \epsilon_p^2 \left(\frac{t_f}{\Theta_f} \right)^{4,1} \frac{M}{L} \left(\frac{p_f \omega}{\Theta^{1,5} t_f} \right)^2 \left(\frac{x_f}{\xi_f} \right)^3 \int_0^{\xi_f} \Theta^{7,1} \xi^2 \, d\xi , \end{aligned} \qquad (22)$$

where X_c, X_e are, respectively, the hydrogen content in the core and in the envelope.

After a series of trials, finally we obtain a satisfactory solar model in which the contents of hydrogen, helium and heavy elements are, respectively, by weight, 0,820, 0,179 and 0,001 in the radiative envelope and in the convective core are, respectively, 0,779, 0,220 and 0,001. The assumed values of density distribution agree very well with the resultant ones ; the differences between the logarithms of the corresponding ones are all within 0,035. They are shown in Table 8. The characteristics of the present solar model are given in Tables 5 and 6.

Table 5. The characteristics of the present solar model

Center			Convective Core $X:Y:Z = 820:179:1$				Radiative Envelope $X:Y:Z = 779:220:1$	
T_c $10^{7\circ}$	ρ_c g/cc	P_c dynes/cm² 10^{17}	$x = r/R$	energy generated $Y_c = L(r)/L$	Mass, $M(r)/M$	Mean M. Wt., μ_c	Mean M. Wt., μ_e	energy generated $(1 - Y_f)$
1,403	1,003	2,000	,0925	0,3061	,05076	0,5800	0,5634	0,6473

Table 6. The various quantities at the interface

ξ_f	P_f 10^{17} dynes/cm²	T_f $10^{7\circ}$	Θ_f	$\left(\frac{d\Theta}{d\xi}\right)_f$	U_{ef}	V_{ef}	U_{cf}	V_{cf}
+0,83485	1,493	1,248	0,88960	−0,25081	+2,71344	+0,57154	+2,7932	+0,58838

From Table 5, we know that the energy generated inside the core amounts 30,6 per cent of the total energy of the sun, while the energy generated in the envelope reaches 64,7 per cent. The sum of them is 95,3 per cent, about 5 per cent less than the observed value. The model with a hydrogen content of 82,5 per cent results overproduction of energy. Unless there is convective core or isothermal occurs, there would not have any appreciable change for the present solar model if the above 5 per cent of energy discrepancy is eliminated. This is clear when one read the run of the values in Table 7, of the characteristics of the three models closest to the present adopted one.

Table 7. Characteristics of 3 models closest to the adopted solar model

H Content, X	Core radius	Energy generated		At the center		
		envelope	core	T_c $10^{7\circ}$	ρ_c g/cc	P_c 10^{17} dynes/cm²
0,800	0,1323	0,3309	0,4476	1,352	82,6	1,680
0,810	0,1197	0,4236	0,4795	1,387	90,6	1,854
0,815	0,1101	0,5020	0,4116	1,389	93,9	1,899

It is well known generally that the error in the energy generation formulae due to the extrapolation of experimental data is much higher than 5 per cent. And the same might be true for the current method to compute the opacity. It is, we think, quite justifiable to regard our result as an equilibrium model.

Since the radiative envelope produces about 65 per cent of the total solar energy, it is not surprise to find the ratio of mean molecular weights in the core to that in the envelope close to 1, namely $\mu_c/\mu_e = 1,029$. This

Table 8.

NUMERICAL INTEGRATION OF THE PRESENT SOLAR MODEL

$\log_{10}\left(\frac{1}{x}-1\right)$	$\log_{10} t$	$\frac{d\log t}{d\log\left(\frac{1}{x}-1\right)}$	$\log_{10} p$	$\frac{d\log p}{d\log\left(\frac{1}{x}-1\right)}$	$\log_{10} q$	$\frac{d\log q}{d\log\left(\frac{1}{x}-1\right)}$	$\log_{10} y$	$\frac{d\log y}{d\log\left(\frac{1}{x}-1\right)}$	$\frac{d\log p}{d\log t}$
—1,000	—1,70357	1,00005	—5,22274	5,05324	—	—0,00002	—	—	5,05299
—0,950	—1,65357	1,00002	—4,97008	5,05324	—	—0,00004	—	—	5,05314
—0,900	—1,60357	1,00002	—4,71742	5,05324	—	—0,00006	—	—	5,05314
—0,850	—1,55357	1,00000	—4,46476	5,05324	—	—0,00010	—	—	5,05324
—0,800	—1,50357	1,00000	—4,21210	5,05324	—	—0,00017	—	—	5,05324
—0,750	—1,45357	0,99998	—3,95944	5,05312	—0,00001	—0,00029	—	—	5,05322
—0,700	—1,40357	0,99995	—3,70679	5,05289	—0,00003	—0,00048	—	—	5,05314
—0,650	—1,35357	0,99988	—3,45415	5,05254	—0,00006	—0,00079	—	—	5,05315
—0,600	—1,30348	1,01498	—3,20154	5,05091	—0,00011	—0,00130	—	—	4,97636
—0,550	—1,25176	1,04971	—2,94942	5,02991	—0,00020	—0,00210	—	—	4,79171
—0,500	—1,19882	1,06522	—2,69878	4,99448	—0,00033	—0,00333	—	—	4,68868
—0,450	—1,14541	1,06965	—2,45008	4,95302	—0,00054	—0,00523	—	—	4,63051
—0,400	—1,09175	1,08418	—2,20352	4,90772	—0,00087	—0,00807	—	—	4,52667
—0,350	—1,03694	1,10448	—1,95957	4,84808	—0,00137	—0,01222	—	—	4,38947
—0,300	—0,98155	1,10910	—1,71885	4,78002	—0,00212	—0,01816	—	—	4,30982
—0,250	—0,92617	1,10494	—1,48162	4,70923	—0,00322	—0,02646	—	—	4,26198
—0,200	—0,87112	1,09618	—1,24794	4,63767	—0,00482	—0,03786	—	—	4,23076
—0,150	—0,81658	1,08515	—1,01785	4,56573	—0,00707	—0,05320	—	—	4,20746
—0,100	—0,76402	1,01967	—0,79116	4,50609	—0,01022	—0,07370	—	—	4, 41917
—0,050	—0,71408	0,98154	—0,56699	4,46190	—0,01456	—0,10108	—	—0,00001	4,54582
0	—0,66561	0,95885	—0,34499	4,41713	—0,02047	—0,13708	—	—0,00002	4,60670

Table 8. (continued)

NUMERICAL INTEGRATION OF THE PRESENT SOLAR MODEL (CONTINUED)

$\log_{10}\left(\frac{1}{x}-1\right)$	$\log_{10} t$	$\frac{d(\log t)}{d\log\left(\frac{1}{x}-1\right)}$	$\log_{10} p$	$\frac{d(\log p)}{d\log\left(\frac{1}{x}-1\right)}$	$\log_{10} q$	$\frac{d(\log q)}{d\log\left(\frac{1}{x}-1\right)}$	$\log_{10} y$	$\frac{d(\log y)}{d\log\left(\frac{1}{x}-1\right)}$	$\frac{d(\log p)}{d(\log t)}$
—	—0,66561	0,95885	—0,34499	4,41713	—0,02047	—0,13708	—	—0,00002	4,60670
+0,050	—0,61808	0,94319	—0,12546	4,36154	—0,02844	—0,18368	—	—0,00005	4,62424
+0,100	—0,57126	0,92974	+0,09086	4,28766	—0,03904	—0,24297	—	—0,00015	4,61168
+0,150	—0,52512	0,91578	+0,30290	4,18938	—0,05297	—0,31702	—0,00001	—0,00044	4,57466
+0,200	—0,47973	0,89937	+0,50931	4,06172	—0,07102	—0,40765	—0,00004	—0,00120	4,51618
+0,250	—0,43525	0,87929	+0,70853	3,90131	—0,09404	—0,51608	—0,00014	—0,00314	4,43689
+0,300	—0,39188	0,85456	+0,89887	3,70629	—0,12294	—0,64261	—0,00040	—0,00773	4,33707
+0,350	—0,34987	0,82473	+1,07861	3,47752	—0,15860	—0,78628	—0,00101	—0,01781	4,21656
+0,400	—0,30949	0,78962	+1,24613	3,21862	—0,20182	—0,94467	—0,00236	—0,03812	4,07616
+0,450	—0,27100	0,74912	+1,40007	2,93596	—0,25325	—1,11378	—0,00511	—0,07537	3,91921
+0,500	—0,23467	0,70349	+1,53947	2,63858	—0,31330	—1,28861	—0,01031	—0,13731	3,75070
+0,550	—0,20074	0,65322	+1,66385	2,33684	—0,38211	—1,46342	—0,01937	—0,23055	3,57742
+0,600	—0,16942	0,59918	+1,77325	2,04113	—0,45955	—1,63279	—0,03394	—0,35805	3,40654
+0,650	—0,14087	0,54270	+1,86822	1,76056	—0,54522	—1,79213	—0,05571	—0,51755	3,24408
+0,700	—0,11517	0,48531	+1,94968	1,50186	—0,63854	—1,93798	—0,08611	—0,70170	3,09464
+0,750	—0,09232	0,42872	+2,01885	1,26926	—0,73877	—2,06828	—0,12611	—0,89993	2,96058
+0,800	—0,07225	0,37456	+2,07707	1,06454	—0,84509	—2,18192	—0,17614	—1,10055	2,84211
+0,850	—0,05480	0,32415	+2,12576	0,88740	—0,95668	—2,27882	—0,23603	—1,29306	2,73762
+0,900	—0,03976	0,27832	+2,16625	0,73631	—1,07270	—2,35896	—0,30517	—1,46876	2,64555
+0,950	—0,02689	0,23757	+2,19978	0,60895	—1,19231	—2,42237	—0,38252	—1,62091	2,56324
+1,000	—0,01592	0,20201	+2,22749	0,50263	—1,31467	—2,46894	—0,46680	—1,74518	2,48814

means that our solar model has little difference with the model which assumes a single value of mean molecular weight throughout the core and the envelope. From the comparison of our result shown in table 5 with those of Naur, and Epstein and Motz, tabulated in table 1, one finds that the hydrogen content of our model is intermediate between theirs. The central temperatures and central density both are a little higher than theirs.

The model of Epstein and Motz who have taken into account of the free-free absorption of hydrogen and computed opacity according to the density and temperature distributions of the solar model should be close to the true state inside the sun and should be comparable to our model. The hydrogen content of their model is much higher, more than 10 per cent, than ours. This may be due partly to (1) the different energy generation laws that we employ a new one for proton-proton reaction which generates energy about 1,4 times slower than theirs, partly to (2) the inconsistence of their opacity laws mentioned above, and partly to (3) the including both of the bound-free and free-free absorptions of hydrogen and helium in our model while only the free-free absorption of hydrogen is considered in theirs.

Table 7 gives the values of numerical integration of the present solar model.

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