Fundamental parameters of pulsating stars from atmospheric models

Szabolcs Barcza

Konkoly Observatory of the Hungarian Academy of Sciences
P.O. Box 67, H-1525 Budapest, Hungary

A purely photometric method is reviewed to determine distance, mass, equilibrium temperature, and luminosity of pulsating stars by using model atmospheres and hydrodynamics. T Sex is given as an example: on the basis of Kurucz atmospheric models and $UBVRI$ (in both Johnson and Kron-Cousins systems) data, variation of angular diameter, effective temperature, and surface gravity is derived as a function of phase, mass $\mathcal{M} = (0.76 \pm 0.09)\mathcal{M}_\odot$, distance $d = 530 \pm 67$ pc, $R_{\rm max} = 2.99 R_\odot$, $R_{\rm min} = 2.87 R_\odot$, magnitude averaged visual absolute brightness $\langle M_V^{\rm mag} \rangle = 1.17 \pm 0.26$ mag are found. During a pulsation cycle four standstills of the atmosphere are pointed out indicating the occurrence of two shocks in the atmosphere. The derived equilibrium temperature $T_{\rm eq} = 7781$ K and luminosity $(28.3 \pm 8.8) L_\odot$ locate T Sex on the blue edge of the instability strip in a theoretical Hertzsprung-Russell diagram. The differences of the physical parameters from this study and Liu & Janes (1990) are discussed.

Introduction

Except for supergiants and cool stars with effective temperature $T_{\rm e} < 4000$K a large grid of model atmospheres is available in the literature (Kurucz, 1997 for the latest versions). The conversion of theoretical fluxes to observational magnitude and colour indices has been solved (e.g. for Johnson and Kron-Cousins photometries, Castelli, 1999) opening the way to a determination of the atmospheric parameters effective temperature, $T_{\rm e}$, and surface gravity if broad band photometry is available for the target star.

The stellar angular diameter can be derived in the following manner from atmospheric models. Monochromatic flux $\mathcal{F}_\lambda(0)$ of the models at optical depth $\tau = 0$ is given in tabular form for a large grid of $T_{\rm e}$, g, metallicity $[M]$, etc., furthermore, by integrating $\mathcal{F}_\lambda(0)$ for a number of photometric systems the fluxes $\mathcal{F}_X(0)$ were computed in physical units $\mathrm{erg\,s^{-1}\,cm^{-2}}$ as well, and the corrections from interstellar absorption, reddening were included (Kurucz, 1997). X represents the photometric band, e.g. V in the Johnson system. On the other hand, observed physical fluxes, $\mathcal{I}_X$, of stars in band X can be derived by using the absolute flux calibration of Vega (Tüg et al., 1977). If the interstellar extinction is A_X magnitude, the half angular diameter of a spherical (i.e. non-distorted) star can be derived from the simple formula

$$\vartheta = R/d = [10^{A_X/2.5}\mathcal{I}_X/\mathcal{F}_X]^{1/2} \tag{1}$$

where d is the distance to the star, R is the radius of $\tau = 0$ (Barcza, 2003).

For giant, main sequence, or white dwarf stars the stellar atmosphere is a thin layer in comparison with R. The gravitational acceleration

$$g(r) = -G\frac{\mathcal{M}}{r^2} = \varrho^{-1}(r)\nabla p(r) \tag{2}$$

is practically constant in the atmosphere, i.e. at $r \approx R$, where $\varrho(r)$, $p(r)$, $\mathcal{M}$, G are density, pressure, mass, and the Newtonian gravitational constant, respectively. The surface gravity, $g(R)$, is an important parameter of the static atmospheric models given conventionally as $\log|g|$ and it can be used to weigh a star if the stellar radius, R, is known. Relation (2) was applied for a number of non-variable stars enhancing our knowledge on stellar masses: e.g. for white dwarfs purely from $\log|g(R)|$ of model atmospheres and R $\mathcal{M}_{\mathrm{wd}} = 0.480 \pm 0.014$ average mass was derived (McMahan, 1989) for a sample of 53 stars while other methods gave somewhat larger mass, $\mathcal{M}_{\mathrm{wd}} = 0.58 \pm 0.10$ for 64 stars (Koester et al., 1979).

Problems, dependence of g, ϑ, $\mathcal{M}$ on $E(B-V), [M]$ were discussed in numerous papers, e.g. Kinman & Castelli (2002), Decin et al. (2003). Accuracy of the data from broad band photometry is inferior to that from fine analysis of spectra or interferometry, respectively, and $\mathcal{M}$ can be determined much better from binary orbits. However, if the more accurate methods are not available, the photometric determination of g, ϑ, and $\mathcal{M}$ must be appreciated.

Implementation for $UBV(RI)_{\mathrm{C}}$ photometry

The main parameters of atmospheric models are T_{e}, g, $[M]$, and in addition to them the interstellar reddening, extinction (e.g. $E(B-V)$, A_X) must be known. From observational point of view colour indices, $C_i, (i = 1, 2, ...)$ are the input parameters and the inverse problem of ordering a model to them must be solved. In optimal cases the typical errors of interpolation are 0.1 in $\log|g|$ while a few hundredth in $\mathcal{F}_X$, respectively, however, the successful solution depends on the shape of the functions $T_{\mathrm{e}}(C_1, ...)$ and $\log|g(C_1, ...)|$. The final step is that theoretical $\mathcal{F}_X(C_1, ...)$ belonging to the selected model must be compared with $\mathcal{I}_X$ from observed magnitudes according to (1).

From inspecting the Kurucz tables some general rules can be inferred, we mention a few of them. For main sequence or giant stars UBV photometry must be preferred if $7000 < T_{\mathrm{e}} < 10^4$ K while for cooler stars BVI_{C} gives reliable results as well. Below $T_{\mathrm{e}} = 9000$ K the dependence of $\log|g|$ on $U-B$ becomes more pronounced, $U-B$ itself is sensitive to $[M]$ because of the appearance of metallic lines in ultraviolet. In favourable cases T_{e} and g can be determined from a two colour diagram if $[M]$, $E(B-V)$ were already known while the procedure is more uncertain if $[M]$ and $E(B-V)$ must be estimated from colour indices as well, in this case more than two colour indices are necessary.

Table 1: Half angular diameter of some stars from intensity interferometry (ϑ_{HB}) and V, $B-V$, $U-B$ of Kurucz models (ϑ), the units are 10^{-9}. The mass was calculated from the *Hipparcos* parallax π (ESA, 1997) and using $R = 3.086 \times 10^{13}\vartheta/\pi$ in (2).

HR	V	$B-V$	$U-B$	ϑ_{HB}	ϑ	π	$\mathcal{M}_\odot$
2421	1.93	0.00	0.04	$3.37 \pm .21$	3.55	0.031	1.51
2491[1]	−1.46	0.00	−0.05	$14.28 \pm .39$	14.31	0.379	3.47
2943[2]	0.38	0.42	0.02	$13.47 \pm .42$	11.80	0.286	1.7
3685	1.68	0.00	0.03	$3.85 \pm .17$	3.89	0.029	4.07
4534	2.14	0.09	0.07	$3.22 \pm .24$	3.33	0.090	1.70
6556	2.08	0.15	0.10	$3.95 \pm .31$	3.79	0.070	2.24
7001	0.03	0.00	−0.01	$7.85 \pm .11$	7.97	0.129	2.47
7557[3]	0.77	0.22	0.09	$7.22 \pm .34$	7.65	0.194	1.18
8728	1.16	0.09	0.08	$5.09 \pm .34$	5.29	0.130	1.55

1: The uncertainty of the mass values is indicated by $3.47_\odot$ for Sirius (HR 2491) in contrast with $2.143_\odot$ from binary orbit (Gatewood & Gatewood, 1978). This severe discrepancy is caused by the steep function $\log|g(U-B, B-V)|$ if $T_{\mathrm{e}} > 9800$K: small shifts $U-B = -0.05 \to -0.04$ or $B-V = 0.00 \to -0.01$ would result in $\log|g| = 4.52 \to 4.3$ i.e. $3.47_\odot \to 2.4_\odot$ with unchanged ϑ.

2: In the effective temperature range of Procyon (HR 2943) ($T_{\mathrm{e}} \approx 6600$ K) the dependence $\Delta\log|g|/\Delta[M] \approx 1.5$ is strong, $[M] = 0.4$ must be taken to reproduce the mass $\mathcal{M} = 1.75$ from binary orbit solution, in spite of the solar atmospheric composition (Drake & Laming, 1995). The change $B-V = 0.42 \to 0.43$ could lead to $\vartheta \approx \vartheta_{\mathrm{HB}}$.

3: The difference of ϑ and ϑ_{HB} exceeds the error for Altair (HR 7557). Its rotational oblateness (Belle et al., 2001) may be responsible because both ϑ and ϑ_{HB} assume spherical symmetry of the visible disc of the star. From $R/d = (ab)^{1/2} = (7.85 \pm 0.27) \times 10^{-9}$ a better agreement could be obtained where a, b are the visible minor and major axes, respectively.

To demonstrate reliability of the method, some date are given in Table 1: ϑ was derived from observed V, $B-V$, $U-B$ (Hoffleit, 1982) by using $\mathcal{F}_V(U-B, B-V)$ of the Kurucz models while for comparison the value ϑ_{HB} is given from intensity interferometry corrected for limb darkening (Hanbury Brown et al., 1974). Using the *Hipparcos* parallaxes (ESA, 1997) $R = 3.086 \times 10^{13}\vartheta/\pi$ km was calculated and the mass was obtained by (2) from $\log|g(U-B, B-V)|$. It can be noted that the weakest sampling of the spectrum is provided by $(U-B, B-V)$ since the wavelength interval $\approx 360 - 540$ nm is covered in this case. Of course an inclusion of other colour indices could improve the values of $\vartheta, \log|g|$.

For pulsating stars with periods exceeding 0.1 day the characteristic time of changes of the main atmospheric parameters $T_{\mathrm{e}}, g_{\mathrm{e}}, \mathcal{F}_X$ is long in comparison with the formation of radiative (and convective) equilibrium, therefore, the static atmospheric models account satisfactorily for the changes of g_{e} and ϑ as a function of pulsation phase φ (Buonaura et al., 1985). The subscript in $g_{\mathrm{e}}(= -\varrho(r)\nabla p(r) > 0)$ indicates that this is an effective outward acceleration produced by the pressure gradient in the atmosphere. On appropriately selected two colour diagrams the loop of a variable star gives $\log g_{\mathrm{e}}(\varphi), T_{\mathrm{e}}(\varphi), \mathcal{F}_X(\varphi)$ etc. of the models while photometric observations can be reduced to $\mathcal{I}_X(\varphi)$ giving finally $\vartheta(\varphi)$ by (1).

- Mass and distance from $\vartheta(\varphi)$, $g_\mathrm{e}(\varphi)$ by using the radial momentum balance in the pulsation

If $\vartheta(\varphi_j), j = 1, 2, ..., N$ points are available in a sufficient number to numerical differentiation of $\vartheta(\varphi)$ the angular acceleration can be converted to $\ddot{R}(t) = \ddot{\vartheta}(t)d$, $t = \varphi P$, where P is the period in seconds. Numerical experience showed that $N \approx 500$ or more points gave reliable $\ddot{R}(\varphi)$ if they were distributed uniformly in $0 \leq \varphi \leq 1$.

Now the main innovation is that $\ddot{R}(\varphi_j)$, $g_\mathrm{e}(\varphi_j), j = 1, 2, ...$ are introduced in the radial component of the Euler equation of hydrodynamics and we exploit that the atmosphere must be in standstill at least twice in a pulsation cycle. At these phases $\varphi_i (i = 1, 2)$ the radial component of the non-linear term vanishes: the non-radial motions are expected to contribute negligibly to the momentum balance in radial direction, therefore, $(\mathbf{v}, \nabla)\mathbf{v} \propto \dot{R} \approx 0$ and we have the equations:

$$\ddot{\vartheta}(\varphi_i)d = -\frac{G\mathcal{M}}{[\vartheta(\varphi_i)d]^2} + g_\mathrm{e}(\varphi_i), \quad i = 1, 2. \tag{3}$$

The fundamental stellar parameters: distance and mass

$$d = \frac{g_\mathrm{e}(\varphi_1) + \ddot{R}_a(\varphi_1) - [g_\mathrm{e}(\varphi_2) + \ddot{R}_a(\varphi_2)][\vartheta(\varphi_2)/\vartheta(\varphi_1)]^2}{\ddot{\vartheta}_0(\varphi_1) - \ddot{\vartheta}_0(\varphi_2)[\vartheta(\varphi_2)/\vartheta(\varphi_1)]^2}, \tag{4}$$

$$\mathcal{M} = [g_\mathrm{e}(\varphi) - \ddot{\vartheta}(\varphi)d]\vartheta^2(\varphi)d^2/G \tag{5}$$

(Barcza, 2003) follow from (3) by elementary operations. The apparent accelerations $\ddot{R}_a$ can be neglected if the characteristic spatial extension of the atmosphere – i.e. the size of the interval $0 \leq \tau_\mathrm{Rosseland} \leq 1$ varies slightly during the whole pulsation cycle. If there are more than two standstills, then $i \geq 3$ in (3). Eventual different values of $\mathcal{M}, d$ from $i = 1, 3$, $i = 2, 3$ etc. can indicate e.g. non-radial pulsation because radial pulsation was assumed throughout the procedure outlined here.

The fundamental stellar parameters equilibrium luminosity and temperature (Carney et al., 1992) are obtained from $\vartheta(\varphi), T_\mathrm{e}(\varphi), d$ by

$$L_\mathrm{eq} = 4\pi\sigma\langle\vartheta^2(\varphi)d^2T_\mathrm{e}^4(\varphi)\rangle, \tag{6}$$

$$T_\mathrm{eq} = \{L_\mathrm{eq}/4\pi\sigma[\langle\vartheta(\varphi)\rangle d]^2\}^{1/4} = \langle\vartheta^2(\varphi)T_\mathrm{e}^4(\varphi)\rangle^{1/4}\langle\vartheta(\varphi)\rangle^{-1/2} \tag{7}$$

where σ is the Stefan-Boltzmann constant, T_eq differs slightly from the average effective temperature $\langle T_e(\varphi)\rangle$. $L_\mathrm{eq}, T_\mathrm{eq}$ allow to locate a variable star in theoretical Hertzsprung-Russell diagram: of course on the basis of [colours $- T_\mathrm{e}, \mathcal{F}_X$] calibration of the used atmospheric models.

— Comparison with the Baade-Wesselink method

The determination of $\vartheta(\varphi)$ is common in both methods. The difference lies in its further use.

In the BW method the radius change is derived from integrating the radial velocity curve and it is equated with ϑd. The kinematic equation

$$\vartheta(\varphi)d = R_0 + \delta R = R_0 + \int_{\varphi_0}^{\varphi P} p_\mathrm{p}(t)[v_\gamma - v_\mathrm{radial}(t)]\mathrm{d}t \tag{8}$$

must be solved for d where $p_{\rm p}$ is the projection factor of converting radial to pulsation velocity and v_γ is the barycentric velocity of the star. Physical input comes from the time-dependent projection factor. A much more serious uncertainty of kinematic nature is imported in this procedure by the error Δv_γ. If $\Delta v_\gamma \ll \overline{|v_\gamma - v_{\rm radial}(t)|}$ a negligible error is propagated into δR, however, it is problematic to achieve this desired accuracy because the observed radial velocities are an integral of the radial component of true, non-uniform motions in the atmosphere contaminated by apparent velocity changes from varying opacity during phases of different compression. The difficulties from the uncertain value of v_γ could be circumvented by differentiating $p_{\rm p}(t)v_{\rm radial}(t)$ and substituting it for $\ddot{\vartheta}d$ in (3). However, because of low number, large scatter of the observed radial velocities, and eventual time dependence of $p_{\rm p}(t)$ this principally correct use of (3) cannot result in $d, \mathcal{M}$ of acceptably small error.

By using $\vartheta, \ddot{\vartheta}, g_{\rm e}$ in (3) radial velocity observations and their problematic conversion to radius changes are not necessary at all at the price of differentiating $\vartheta(\varphi)$ twice plus more physical input: $g_{\rm e}(\varphi)$ and the assumption of radial momentum balance must be used. Finally d and $\mathcal{M}$ are obtained by solving algebraic equations. The method was described in detail and applied for the $\mathrm{RR_{ab}}$ star SU Dra by Barcza (2003). We mention two remarkable details from this study. (i) At minimum radius, i.e. at maximum compression in the atmosphere, $g_{\rm e} = 50.1\,{\rm ms}^{-2} \gg G\mathcal{M}/R^2 \approx 8\,{\rm ms}^{-2}$ reduces (3) to $d < g_{\rm e}/\ddot{\vartheta}$ giving $d <$ 718pc. This upper value is very close to $d =$ 647pc from the solution of (3) for three observed standstills. From them the unequivocal $d, \mathcal{M}$ show that a non-radial mode, if there is any, is negligible in comparison with the radial oscillation of SU Dra. (ii) During a time interval t the small uncertainty $\Delta v_\gamma \approx +5.9{\rm kms}^{-1} \approx 0.04 v_\gamma$ (Liu & Janes, 1990 vs. Oke et al., 1962) leads to a phase dependent error $\Delta R = p_{\rm p}\Delta v_\gamma t \leq 3.43 \times 10^5{\rm km}\,(\approx (R_{\rm max} - R_{\rm min})/2!)$ which has a considerable effect on d. If the method by Liu & Janes (1990) is followed to solve the kinematic equation, the increment of d is 1.26 from $\Delta v_\gamma = 5.9$ km s^{-1}. This results in correction $\langle M_V^{\rm mag}\rangle = 0.78 \to 0.28$ for the magnitude average absolute magnitude. (Roughly this is the difference between the short and long extragalactic distance scales – Gratton, 1998. The distance 815 pc corresponding to the long distance scale is ruled out by $d <$ 718pc.)

Application to T Sex

Now we apply the method for the RRc variable T Sex. Good quality photoelectric observational material was collected from the literature, the sources are given in Table 2.

- Period, $O - C$, average light and colour curves

The periods given by TS58, PP64, EE73, BM88, LJ89 indicated period changes of O(10^{-6}). To clarify it string length minimization (SLM) was applied, since good conversion of the observations to average light curve and colour-colour loops is crucial to determine $\vartheta(\varphi), g_{\rm e}(\varphi)$. (The adaptation of SLM to pulsating stars was described by Barcza (2002), the notation of this paper will be used here. The essence of SLM is that first the segments $k = 1, 2, ...$ of the observed V magnitudes are projected onto the phase axis $\varphi, 0 \leq \varphi \leq 1$ by the saw tooth function which is perturbed by a term $\delta_k \equiv (O-C)_k \ll P_0$ accounting for the phase shift of segment k and next the neighbouring points are con-

Table 2: $V, UBV, BVRI, UBV(RI)_C$ data of T Sex used for constructing average light and colour curves, k is the serial number of the segment, n_k is the number of V points in the segment, δ_k, E_k are the phase shift and epoch in (9)

k	HJD − 2400000	n_k		δ_k	E_k	Source
1	34311.8675 − .9315	31	V	−0.0160	6	TS58
2	34350.8051 − .9402	66	V	−0.0247	126	TS58
3	34363.7245 − .8531	64	V	−0.0300	166	TS58
4	34508.5948 − .6598	10	V	−0.0766	612	TS58
5	35190.7392 − .8208	2	UBV	0.0593	2712	TS58
6	35191.6423 − .8871	72	UBV	0.0586	2715	TS58
7	35195.6440 − .9000	106	UBV	0.0470	2727	TS58
8	35513.7227 − .9620	40	UBV	−0.0286	3707	TS58
9	35514.6815 − .9477	60	UBV	−0.0275	3710	TS58
10	35516.6802 − .9387	73	UBV	−0.0272	3716	TS58
11	38017.9584 − 8.0870	43	UBV	0.0322	11417	PP64
12	38035.8141 − .9985	55	UBV	0.0280	11472	PP64
13	38038.8007 − .8928	28	UBV	0.0227	11481	PP64
14	40678.6990 − .7931	10	V	0.0778	19610	EE73
15	40680.6390 − .7382	8	V	0.0757	19616	EE73
16	41013.7358 − .7565	4	V	−0.0127	20641	EE73
17	43525.792 − 56.694	11	V	0.0588	28418	E94
18	45387.7850 − .9212	76	$BVRI$	0.0010	34109	BM88
19	45388.6578 − .7634	69	$BVRI$	0.0026	34112	BM88
20	45389.6382 − .8760	113	$BVRI$	0.0029	34115	BM88
21	45393.7134 − .8596	98	$BVRI$	0.0000	34127	BM88
22	45400.6689 − .7235	45	$BVRI$	−0.0021	34149	BM88
23	46845.6886 − .7489	6	$UBV(RI)_\mathrm{C}$	0.0540	38598	LJ89
24	46846.8005 − .8859	7	$UBV(RI)_\mathrm{C}$	0.0429	38601	LJ89
25	46847.6521	1	$UBV(RI)_\mathrm{C}$	0.0480	38604	LJ89
26	46848.6547 − .9614	25	$UBV(RI)_\mathrm{C}$	0.0432	38607	LJ89
27	47197.7625 − 8.0035	16	$UBV(RI)_\mathrm{C}$	−0.0301	39682	LJ89
28	47226.6368 − .9153	3	VR_C	−0.0258	39771	LJ89
29	47488.8955 − .9059	7	$BVRI$	−0.0881	40578	BM92

nected by straight strings. Finally the normalized sum νl of the string lengths is minimized numerically as a function of period P_0 and phase shifts $\delta_1, ...$)

The V file contains 1149 observations in 29 segments, SLM gave for the maxima

$$\mathrm{HJD}_{\max} = 2434310.035 + P_0 E_k + \delta_k \tag{9}$$

with $P_0 = 0.3247796 \pm 0.0000032$ and the values δ_k in Table 2. The summed string length was $\nu l_0(P_0, \delta = \mathbf{0}) = 0.330$ with standard deviation SD $= 0.069$ magnitude, the folded light curve belonging to it is plotted in *Fig. 1a*. Applying the values δ_k of Table 2 reduced $\nu l(P_0, \delta)$ to 0.062 with $\mathrm{SD}(P_0, \delta) = 0.012$ mag, dots of *Fig. 1b* are a plot of the folded light curve, its SD $= 0.012$ does not exceed the expected random error of a V point, it is lower than the claimed amplitude 0.028, 0.015 mag for the second and third periods of T Sex (Hobart et al., 1991). Thus, the conclusion must be drawn that between HJD 2434311-2447488 the light curve of T Sex can perfectly be reproduced if the light curve segments in *Fig. 1a* are shifted to the $\mathrm{HJD}_{\max}$ given by (9). The colour indices were shifted by δ_k of the segment, *Figs. 1c-f* are their plots. The line in *Figs. 1b-f* was obtained by fitting high order (≤ 9) polynomials to the points. These drawn light and colour curves were used to construct colour-colour loops and to interpolate the physical parameters of the Kurucz models.

Two remarks on the homogenization of the colour curves. (1) In $U - B$ of segments 5,6,7 a zero point correction +0.08 was applied while for 8,9,10 it was +0.05 in order to bring the TS58 observations in coincidence with those of PP64 and LJ89. (2) The Johnson $V - R, V - I$ colours of BM88 and BM92 were converted to $V - R_\mathrm{C}, V - I_\mathrm{C}$ by Taylor's (1986) empirical formulae.

- *The physical parameters*

The colour-colour loops $(U-B, B-V), (U-B, V-R_\mathrm{C}), (U-B, V-I_\mathrm{C}), (B-V, V-I_\mathrm{C})$ could be used to interpolate $T_\mathrm{e}(\varphi)$, $\log g_\mathrm{e}(\varphi)$, $\mathcal{F}_V(\varphi)$, $\mathcal{F}_{R_\mathrm{C}}(\varphi)$, $\mathrm{BC}(\varphi)$. Using a midpoint formula the half angular velocity $\dot{\vartheta}$ and acceleration $\ddot{\vartheta}$ were determined from $\vartheta(\varphi)$. *Fig. 2* is a plot of the results for $E(B-V) = 0.095, [M] = -1.2$. From the different loops the scatter of $\log g_\mathrm{e}$ was the largest: $\pm 0.11, \pm 0.03$ at $\varphi = 0.3, 0.9$, respectively, the scatter in $\mathcal{F}_V, \mathcal{F}_{R_\mathrm{C}}, T_\mathrm{e}$ was ≤ 0.02. Therefore, $\log g_\mathrm{e}, \vartheta, T_\mathrm{e}$ were averaged at each φ from the four loops. The scatters result in an error of ≈ 0.15 for the derived mass and distance if one pair of standstills is used.

- *Distance, mass, equilibrium luminosity and effective temperature*

The standstills of the atmosphere were found at $\varphi \approx 0.31, 0.56, 0.65, 0.90$ by searching for zero average angular velocity $\overline{\dot{\vartheta}(\varphi)} \approx 0$. $\ddot{\vartheta}, \log g_\mathrm{e}$ were averaged here in an interval $\Delta\varphi \approx 0.02$ (i.e. ≈ 10 min). Table 3 reports the results for some values of $E(B-V) = 0.095, [M] = -1.2, -1.0$.

The characteristic size of the atmosphere is $\approx 10^4$ km, $\ddot{R}_a \ll g_\mathrm{e}$, therefore, $\ddot{R}_a$ can be neglected.

$\langle T_\mathrm{e}(\varphi)\rangle, T_\mathrm{eq}$ and from pairs of standstills AD,BD,CD the average d, $\mathcal{M}$, R, L_eq are given in Table 4 to some values of $E(B-V)$. The estimated errors from the averaging are in accordance with that of the interpolation of $\log g_\mathrm{e}$, the other quantities propagate negligible errors in (4),(5).

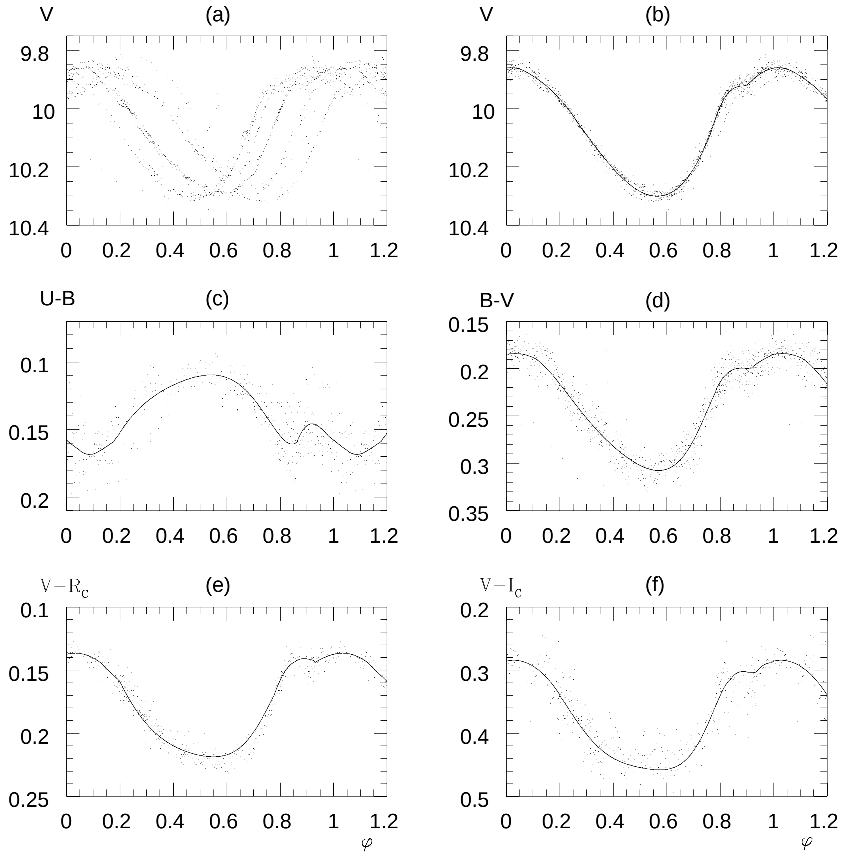

Figure 1: Panel (a): 1149 V observations folded by $P_0 = 0.3247796$ days, $\delta = \mathbf{0}$. Panels (b)-(f): dots: $V, U-B, B-V, V-R_{\rm C}, V-I_{\rm C}$ observations folded with the values δ_k given in Table 1. Lines: High order polynomial fitting curves.

Table 3: Average ϑ, $\ddot{\vartheta}$, $\log g_{\rm e}$ at $\varphi = 0.31,\ 0.56,\ 0.65,\ 0.90$ for $E(B-V) = 0.095, [M] = -1.2, -1.0$. The units are: radian, radian $\rm s^{-2}$, $\rm cm\,s^{-2}$.

	φ	$\vartheta \times 10^{10}$	$\ddot{\vartheta} \times 10^{18}$	$\log g_{\rm e}$
$[M] = -1.20$				
A	0.31	1.273	$-0.334 \pm .013$	$3.38 \pm .11$
B	0.56	1.242	$0.189 \pm .012$	$3.37 \pm .10$
C	0.65	1.247	$-0.257 \pm .020$	$3.36 \pm .07$
D	0.90	1.222	$1.56 \pm .02$	$3.71 \pm .03$
$[M] = -1.00$				
A′	0.31	1.264	$-0.331 \pm .013$	$3.45 \pm .10$
D′	0.90	1.217	$1.58 \pm .02$	$3.74 \pm .03$

Table 4: Average physical quantities from the pairs of standstills AD,BD,CD assuming $M = -1.20$ and different reddenings. The units are: K, K, pc, solar mass, radius, and luminosity. Our final choice is $E(B-V) = 0.095$.

$E(B-V)$	$\langle T_e \rangle$	T_{eq}	d	$\mathcal{M}$	$\langle R \rangle$	L_{eq}
0.08	7607	7625	452 ± 73	$0.37 \pm .06$	2.54	19.6 ± 6.9
0.09	7712	7728	514 ± 84	$0.55 \pm .08$	2.86	26.5 ± 9.4
0.095	**7765**	**7781**	$\mathbf{530 \pm 67}$	$\mathbf{0.76 \pm .09}$	**2.93**	$\mathbf{28.3 \pm 8.8}$
0.10	7816	7834	558 ± 65	$1.06 \pm .16$	3.07	31.9 ± 6.9
0.105	7871	7886	892 ± 130	$3.04 \pm .44$	4.89	84 ± 26

Discussion

- *Dependence of the physical quantities on reddening*

The strong dependence of $T_e(\varphi), \log g_e(\varphi)$ and the derived $d, \mathcal{M}$ on $E(B-V)$ is a surprising result of this study while the value of $[M]$ is of secondary importance. $E(B-V) = 0.05 \pm 0.02$ by Liu & Janes (1990) gives very small $d, \mathcal{M}$ which cannot be reconciled with our present day theoretical knowledge on RR Lyrae or other type of pulsating stars. Reddening $E(B-V) = 0.07$ and 0.09 were suggested by Hobart et al. (1991) and Hemenway (1975), respectively, our finding is that $E(B-V) = 0.09 - 0.1$ is the only possible choice.

- *Effective temperature, surface gravity*

$T_e(\varphi), \log g_e(\varphi)$ of the present study are significantly higher by some 650 K and ≈ 0.4 than those of Liu & Janes (1990). Since essentially the same Kurucz tables were used we attribute the difference to the different philosophy of the interpolation procedure:

> Liu & Janes (1990) determined $g_e(\varphi) = 0.6\mathcal{M}_\odot G/R^2(\varphi)$ from kinematics with assuming the canonical mass $0.6\mathcal{M}_\odot$ of RR Lyrae stars. Next, one arbitrarily chosen colour index, $V-K$ was taken as sole source of $T_e(\varphi)$ belonging to $\log g_e(\varphi)$ from the kinematics and $T_e(\varphi)$ from the other colour indices were neglected leading to their very low $\langle T_e \rangle = 7137$ K. To check this procedure their $T_e(B-V), T_e(V-R_C), T_e(V-I_C), T_e(V-K)$ tables were all used for $\log g_e = 3, 3.5$: according to our opinion no reason can be found to reject any T_e since the difference was ≤ 250 K with no systematically decreasing trend when going more and more to the infrared indices. Therefore, averaging could have been more appropriate. Theoretical colour indices differing systematically from the observed ones by $0.02 - 0.04$ mag correspond to their lower $\log g_e(\varphi)$ values.
>
> In the present study two colour diagrams were used to determine a pair of $T_e(\varphi)$, $\log g_e(\varphi)$ simultaneously taking into account the dependence of $T_e(\varphi)$ on $\log g_e(\varphi)$ automatically and the small differences (≤ 250 K in T_e, ≤ 0.11 in $\log g_e$) from the four colour-colour loops justified averaging. This is a self-consistent procedure trusting on the Kurucz models exclusively. By using four colour-colour loops, different parts of the whole spectrum $\mathcal{F}_\lambda(0)$ were sampled in the λ interval $360 - 1000$ nm.

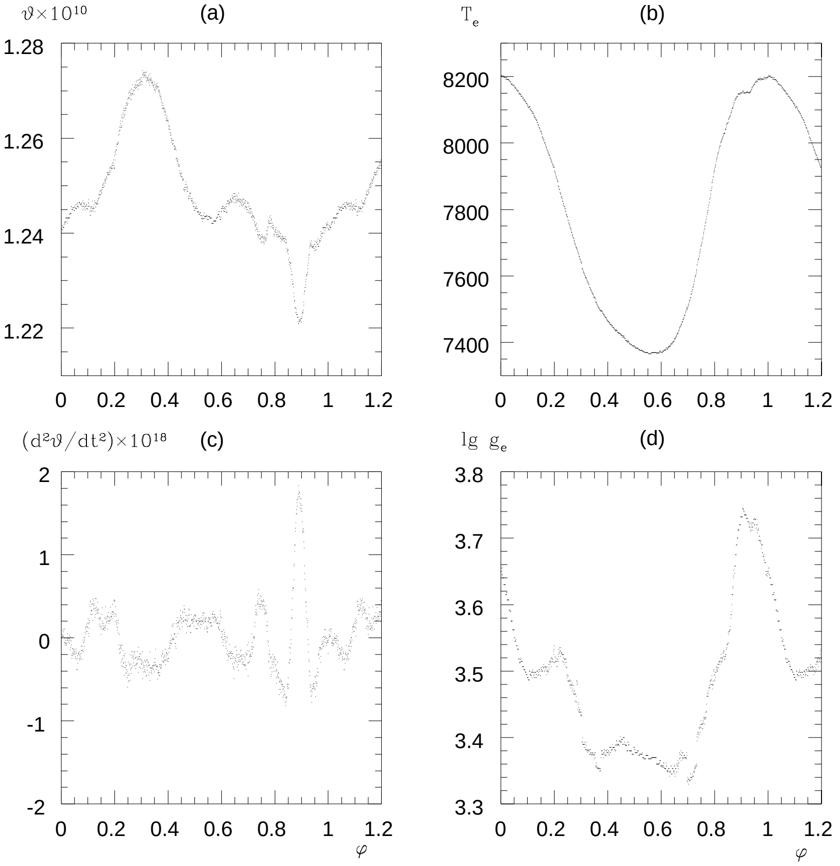

Figure 2: The average variable parameters of T Sex for $E(B-V) = 0.095, [M] = -1.2$ from the loops $(U-B, B-V), (U-B, V-R_C), (U-B, V-I_C), (B-V, V-I_C)$. The outlier points of panels (a,c,d) at $\varphi \approx 0.36, 0.72-0.76$ are artifacts of interpolation, they were not smoothed out to show the effect of eventual uncertainties in the interpolation.

- *Position in a theoretical HRD, absolute brightness*

The values of the fundamental parameters $\log T_{\rm eq} = 3.891$, $\log L_{\rm eq} = 1.452$, $\mathcal{M} = 0.76_\odot$ for $E(B-V) = 0.095$ place T Sex just to the blue edge of the instability strip given by Tuggle & Iben (1972) and Lee et al. (1990). This position is expected for an RR_c variable and it was our main argument to accept the large reddening. With distance $d = 530$ pc we get $R(\varphi = 0.31) = 2.99R_\odot, R(\varphi = 0.91) = 2.87R_\odot$, and magnitude averaged visual absolute brightness $\langle M_V^{\rm mag}\rangle = 1.17 \pm 0.26$ mag. These values come purely from atmospheric models which are trustable to a few percent level and verify figures from stellar structure calculations. The values $d = 578.5$ pc and $\langle M_V^{\rm mag}\rangle = 1.06 \pm 0.38$ of Hobart et al. (1991) agree with the present results within the quoted errors. The physical parameters of Liu & Janes (1990), ($0.47\mathcal{M}_\odot$, $d = 667$ pc, $\langle T_{\rm e}\rangle = 7137$ K, $\langle R\rangle = 4.05R_\odot$, $\langle M_V\rangle = 0.76 \pm 0.27$ mag $\langle \log g_{\rm e}\rangle = 2.98$) are unreliable.

Figs. 3a-c are plots of the variable acceleration, velocity, radius in absolute units for a pulsation cycle if $d = 530$ pc. The sharp undulation of the curves in $0.72 < \varphi < 0.78$ is not real, it originates from the interpolation artifact indicated in the caption to *Fig. 2*.

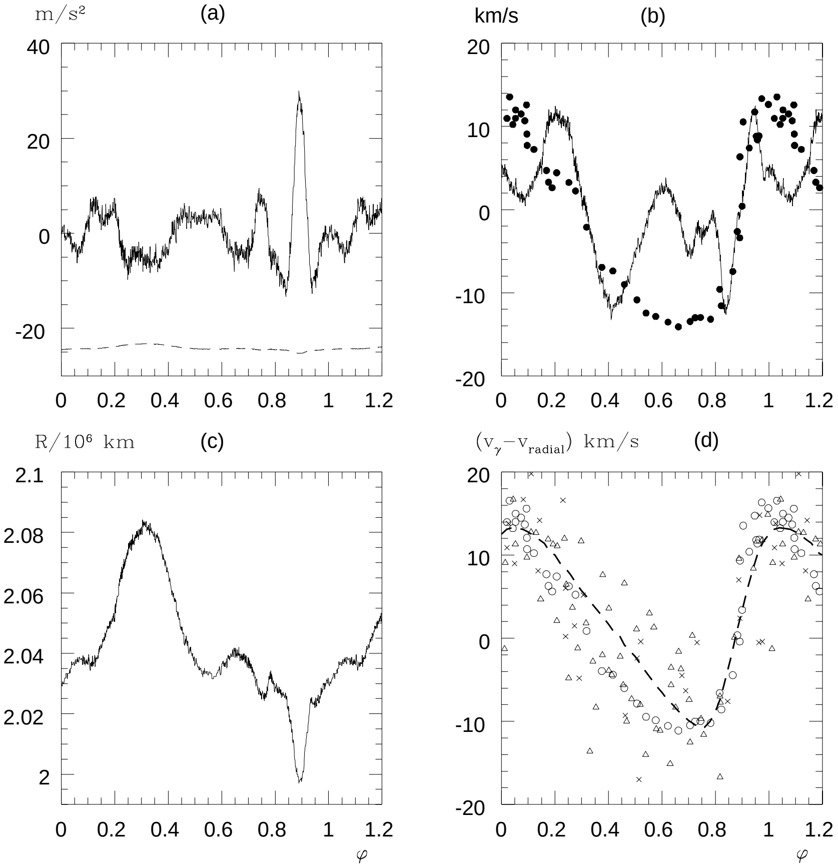

Figure 3: Panel (a): line: $g_e(\varphi)$, dashed: $-0.76\mathcal{M}_\odot G/R^2(\varphi)$. Panel (b): $\dot{R}(\varphi)$, filled circles: $25 - v_{\mathrm{radial}}(\varphi)$ from Liu & Janes (1989). Panel (c): $R(\varphi)$. Panel (d): circles: $28 - v_{\mathrm{radial}}(\varphi)$ from Liu & Janes (1989), triangles: $38 - v_{\mathrm{radial}}(\varphi)$ from Barnes et al. (1988), crosses: $38 - v_{\mathrm{radial}}(\varphi)$ from Tifft & Smith (1958), dashed line: mean of $38 - v_{\mathrm{radial}}(\varphi)$ from Barnes et al. (1988). The outlier points of panels (a)-(c) at $0.72 < \varphi < 0.78$ are artifacts of interpolation, they were not smoothed out to show the effect of eventual uncertainties in the interpolation.

- *Open problems*

There is a systematic difference in the $v_{\mathrm{radial}}(\varphi)$ curves of Barnes et al. (1988), Tifft & Smith (1958), and Liu & Janes (1989): $v_\gamma = 38$, and 28.0 ± 1.4 km s^{-1}, respectively, which exceed the expected observational errors. On the basis of *Fig. 3b* $v_\gamma = 25$ km s^{-1} seems even more probable. The amplitudes $28, 24, 24$ km s^{-1} of TS58, BM88, LJ89 do not differ significantly. *Fig. 3d* shows that subtracting $v_{\mathrm{radial}}(\varphi)$ from the different barycentric velocities does not bring the different observations to full coincidence. The stability of the light and colour curves is, however, obvious from *Fig. 1*, therefore, secular change of the shape of $v_{\mathrm{radial}}(\varphi)$ seems to be improbable. A variable v_γ i.e. duplicity cannot be excluded in spite of the large scatter of $v_{\mathrm{radial}}(\varphi)$ of TS58, BM88, furthermore, the scatter of δ_k is some 3 hours. This is significantly higher than the imaginable error of a δ_k value and for its explanation barycentric motion i.e. light-time effect can even be considered.

On speculation level the extreme sensitivity of the BW method on Δv_γ may be guessed as the main source of the discrepant d of Liu & Janes (1989) which was propagated into their $\mathcal{M}$, $\langle R\rangle$, $\langle M_V\rangle$. Since the radial amplitude of T Sex is only some $\delta R = 80000$ km for a BW analysis v_γ ought to be known by an accuracy O(100)m s^{-2} which was obviously not reached, another factor is their low $T_\mathrm{e}(\varphi)$. (Their quoted error $\Delta v_\gamma = \pm 1.4$ km s^{-1} leads to an error $\approx \pm 18000$ km in δR resulting in an error $\Delta d/d \approx 0.21$.)

The product $p_\mathrm{p}[v_\gamma - v_\mathrm{radial}(\varphi)]$ is the comparable quantity with $\dot{R}$ plotted in *Fig. 3b*. The spectral lines originate from $0 < \tau_\mathrm{Rosseland} < 0.4$, therefore, we expect $p_\mathrm{p}|v_\gamma - v_\mathrm{radial}(\varphi)| < |\dot{R}(\varphi)|$. It is satisfied if $p_\mathrm{p}(t) \approx 1$, however, the large scatter of the radial velocities gives weak basis for this very small value. Remarkable is that the most accurate radial velocities i.e. those of Liu & Janes (1989) show small humps at the extreme values of $\dot{R}$ at $\varphi \approx 0.2, 0.4, 0.6$ and $\dot{R}_\mathrm{max} - \dot{R}_\mathrm{min}$ agrees better with the extreme values of $v_\gamma - v_\mathrm{radial}$ if $v_\gamma = 25$ km s^{-1}.

The rather loose correlation of $\dot{R}$ and $25 - v_\mathrm{radial}(\varphi)$ is similar to that found in SU Dra (Barcza, 2003) and it raises a serious question concerning the basic equation (8) of the BW method. A qualitative explanation can be guessed from gas dynamics and the technique of measuring radial velocity.

> $\dot{\vartheta}(\varphi), \dot{R}(\varphi)$ reflect the motion of $\tau = 0$ while the spectral lines originate from the surroundings of $\tau_{\lambda_\mathrm{line}} \approx 0.3$. Non-negligible velocity gradient is definitely present in an RR Lyrae atmosphere (e.g. Oke et al., 1962) and the limb darkening integrates the non-uniform motion of the layers $0 < \tau < 0.5$ into a single value $v_\mathrm{radial}(\varphi)$. (Dynamical atmospheric models are not available to treat quantitatively the conversion of the pulsation velocity to $v_\mathrm{radial}(\varphi)$.)
>
> CORAVEL technique is itself accurate for stars of non-variable spectra while applying it for a variable spectrum may result in systematic errors which are not easy to survey. The coarse agreement of $v_\mathrm{radial}(\varphi)$ from spectroscopy and CORAVEL is obvious in *Fig. 3d*, however, the large scatter indicates that some caution is appropriate, especially, since fine details of $\vartheta(\varphi)$, $v_\mathrm{radial}(\varphi)$ play some role in a BW analysis.

Conclusions

The purely photometrically derived fundamental parameters of T Sex (see Table 4) have been found from ATLAS atmospheric models of Kurucz (1997) and their calibration to stellar photometric systems (Castelli, 1999). They have been found to be in consensus with our knowledge on stellar models and pulsation theory of asymptotic giant branch stars. In addition to bridging over these remote branches of astrophysics some details have been revealed on RR Lyrae type pulsation: at the RRc variable T Sex fine structure, definite footprint of two shocks have been found in the variable stellar radius $R(\varphi)$ (i.e. in the distance of zero optical depth from the stellar centre). In a previous study of SU Dra similar details were found concerning the fine structure of the atmospheric pulsation: there is at least one pair of temporal, intermediate, minor standstills of the pulsating atmosphere between maximum and minimum extension. This seems to be a common feature of RRab and RRc stars at phase ≈ 0.55, it was not considered (or it was smoothed out) in the previous BW studies, presumably because it is a sub-oscillation in the upper stellar atmosphere which is scarcely reflected in the radial velocities. (The radial velocities give information on the motion of the deeper layers.) To derive the fundamental parameters the less accurate radial velocity observations and their problematic conversion to radius changes had not to be used at all, however, an indication of eventual variable barycentric velocity of T Sex has arisen.

Acknowledgements This research has made use of SIMBAD database operated at CDS, Strasbourg, France and ADS of NASA. The author is grateful to J. M. Benkő for comments on the text.

References:

Barcza S., 2002, *A&A*, **384**, 460
Barcza S., 2003, *A&A*, **403**, 683
Barnes T. G. III, Moffett T. J., Hawley S. L., Slovak M. H., & Frueh, M. L., 1988, *ApJS*, **67**, 403 (BM88)
Barnes T. G. III, Moffett T. J., & Frueh M. L., 1992, *PASP*, **104**, 514 (BM 92)
Belle van G. T., Ciardi D. R., Thompson R. R., Akeson R. L., & Lada E. A., 2001, *ApJ*, **559**, 1155
Buonaura B., Caccin B., Onnembo, A, Russo G., & Sollazzo C., 1985, *Mem. Soc. Astron. Ital.*, **56**, 153
Carney B. W., Strom J., & Jones R. V., 1992, *ApJ*, **386**, 663
Castelli F., 1999, *A&A*, **346**, 564
Decin L., Vandenbussche B., Waelkens K., Eriksson C., Gustafsson B., Plez B., & Sauval A. J., 2003, *A&A*, **400**, 695
Drake J. & Laming M., 1995, *The Observatory*, **115**, 118
Eggen O. J., 1994, *AJ*, **107**, 1834 (E94)
Epstein I. & Epstein A. E. A., 1973, *AJ*, **78**, 83 (EE73)
ESA, 1997, *The Hipparcos Catalogue*, ESA SP-1200
Gatewood G. D. & Gatewood C. V., 1978, *ApJ*, **225**, 191
Gratton R. G., 1998, *MNRAS*, **296**, 739
Hanbury Brown R., Davis J., & Allen L. R., 1974, *MNRAS*, **167**, 121
Hemenway M. K., 1975, *AJ*, **80**, 199
Hobart M. A., Peña J. H., & Peniche R., 1991, *Rev. Mexicana Astr. Ap.*, **22**, 275
Hoffleit D., 1982, *The Bright Star Catalogue*, Yale Univ. Obs.
Kinman T. & Castelli F., 2002, *A&A*, **391**, 1039
Koester D., Schulz H., & Weidemann V., 1979, *A&A*, **76**, 262
Kurucz R. L., 1997, `http://cfaku5.cfa.harvard.edu`
Lee Y.-W., Demarque P., & Zinn R., 1990, *ApJ*, **350**, 155
Liu T. & Janes K. A., 1989, *ApJS*, **69**, 593 (LJ89)
Liu T. & Janes K. A., 1990, *ApJ*, **354**, 273
McMahan R. K., 1989, *ApJ*, **336**, 409
Oke J. B., Giver L. P., & Searle L., 1962, *ApJ*, **136**, 393
Preston G. W. & Paczyński B., 1964, *ApJ*, **140**, 181 (PP64)
Taylor B. J., 1986, *ApJS*, **60**, 577
Tifft W. G. & Smith H. J., 1958, *ApJ*, **127**, 591 (TS58)
Tuggle R. S. & Iben I. Jr, 1972, *ApJ*, **178**, 455
Tüg H., White N. M., & Lockwood G. W., 1977, *A&A*, **61**, 679