SML: a new program for photometric spot modelling

M A N U A L

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1 Introduction: analytic spot models

In the analytic spot model of Budding [1] or Dorren [2] it is assumed that the loss of light is caused by a circular spot with center (λ, β) and with angular radius γ. The radius of the star is the unit length. See Fig. 1 for more details. The main results of these models are:

1. The loss of light due to the presence of the spot depend only on the γ size of the spot, and on the ε angle between the surface normal at the spot center and the line of sight.

2. The equation determining the angle ε (Fig. 2) at time t :

$$z_0 = \cos\varepsilon = \cos(\lambda - \Phi)\cos\beta\sin i + \sin\beta\cos i, \tag{1}$$

   where i is the inclination of the rotation axis to the line of sight, and Φ is the phase of rotation defined by:

$$\Phi = 2\pi\,\frac{t-E}{P}, \tag{2}$$

   where E is the epoch and P is the rotational period of the star.

3. Budding [1] defined σ integrals in the following general form:

$$\sigma_n^m = \frac{1}{\pi}\iint_{\text{spot's area}} x^m z^n \,\mathrm{d}x\mathrm{d}y. \tag{3}$$

   This xyz rectangular coordinate system is related to a spherical polar system (λ, β) to express coordinates on a stellar surface.

- The integral σ_0^0 is the projected spot's area. It is the most important quantity in the description of the light loss.
- The integral σ_1^0 describes the effect of the linear limb darkening.

4. Depending on the position of the spot, there are three possible cases:
   - the entire outline of the spot is visible: $\varepsilon \leq \frac{\pi}{2} - \gamma$.
   - the spot is partially in view: $\frac{\pi}{2} - \gamma < \varepsilon < \frac{\pi}{2} + \gamma$.
   - the spot is out of view: $\varepsilon \geq \frac{\pi}{2} + \gamma$. In this case: $\sigma_0^0 = \sigma_1^0 = 0$.
5. Budding and Dorren developed analytic formulae for the σ_0^0 and σ_1^0 integrals. The explicit forms of these integrals are given in [1] and [2].
6. By introducing the linear limb darkening coefficient u, Budding defined the following spot-darkening function [1]:
$$\sigma_c(u, \gamma, z_0(t)) = \frac{3}{3-u} \left\{(1-u)\,\sigma_0^0 + u\,\sigma_1^0\right\}. \tag{4}$$
7. Using these notations, the intensity of the spotted star is:
$$I_c(t) = U\left\{1 - (1-\kappa_w)\sigma_c(u, \gamma, z_0(t))\right\}, \tag{5}$$
where U is the unspotted intensity of the star, and the coefficient κ_w is the flux ratio of the spot and the unspotted photosphere depending on the wavelength and on the relative temperature of the spot to the photosphere.
8. In case of binary systems, the loss of light depends on the L_1 intensity of the spotted star and on the U intensity of the system:
$$I_c(t) = U - L_1(1-\kappa_w)\sigma_c(u, \gamma, z_0(t)). \tag{6}$$
9. If USI denotes the unspotted magnitude of the star, then the brightness of the spotted star in magnitude is:
$$l_c(t) = \mathrm{USI} - 2.5\,\log_{10}\left\{\frac{U - L_1(1-\kappa_w)\sigma_c(t)}{U}\right\}. \tag{7}$$

These formulae can be extended in a straightforward manner if the number of spots is greater than one.

1.1 The standard spot model

This is the original model proposed by Budding [1]. In this case the observed $l_o(t)$ intensities are fitted by the theoretical intensity function $l_c(t)$ in the whole time interval. For this purpose a nonlinear least squares algorithm (like the Marquardt-Levenberg algorithm) is needed. By minimizing the sum of squared residuals: $\chi^2 = \sum_{i=1}^{m} \frac{(l_{o_i} - l_{c_i})^2}{\Delta l_i^2}$, we get the optimal parameter values. In this model, the spot parameters are constant.

1.2 Time series spot model

In some cases the shape and amplitude of the light curve change in function of time. This is due to the movement, growth or decay of the spots. In this model the time interval is divided into smaller subintervals and for each interval the standard model is applied. This way we get a set of spot parameters in function of time. The theoretical intensity curve is obtained by averaging the theoretical intensity values of the standard models in the overlapping intervals. This procedure was developed and implemented first by Bartus ([3]) based on an earlier more simple approach by Strassmeier & Bopp ([4]).

1.3 Spot temperature determination

The theoretical intensity function $l_c(t)$ depends on the wavelength Λ of the observing instrument through the parameter u and κ_w, and depends on the spot temperature through the parameter κ_w. In this program the following method is used for the determination of the spot temperature:

1. the light curve is observed in multiple bands.

2. the energy spectrum of the star is approximated by the Planck function, and the flux ratio between the spot and the photosphere is calculated by:

$$\kappa_w(\Lambda, T_{\mathrm{star}}, T_{\mathrm{spot}}) = \frac{\exp\left(\dfrac{hc}{k\Lambda T_{\mathrm{star}}}\right) - 1}{\exp\left(\dfrac{hc}{k\Lambda T_{\mathrm{spot}}}\right) - 1}. \tag{8}$$

3. by using different u and USI values for the different bands and by inserting eq. 8 into eq. 7, we get the multicolor theoretical intensity function. The sum of squared residuals of the multicolor data is defined as:

$$\chi^2 = \sum_i \chi^2(\Lambda_i), \tag{9}$$

   where $\chi^2(\Lambda_i)$ is the sum of squared residuals of the data in the band Λ_i. The spot parameters and the spot temperature T_{spot} are refined using the multicolor observed data and the multicolor theoretical intensity functions.

Previously, see Strassmeier & Oláh [5], a similar method was used but the spot temperature was determined from the $\chi^2(T_{\mathrm{spot}})$ curve. The novelty of the new method is the direct refinement of the parameter T_{spot} through the nonlinear eq. 8. Note that the program can be easily modified by replacing the function $\kappa_w(\Lambda, T_{\mathrm{star}}, T_{\mathrm{spot}})$ with other functions that may better describe the stellar flux distribution.

2 Description of the new program SpotModeL

The basic purpose of the program is to determine the geometrical and physical spot parameters using the standard spot model. Alternatively time series analysis can be performed. If multicolor data is available, the spot temperature can be determined too.

The parameters of the theoretical intensity function, defined by eqs. 1, 2, 4, 7 and 8, are refined using the analytic spot model of Budding [1] or Dorren [2] respectively. The limit for the number of spots is three.

SpotModeL uses magnitude as unit for input/output data, model light curves and brightness parameters.

2.1 Software environment

The program was developed under UNIX operating systems. It is based on a modified version of the program *gnuplot* [7], which is suitable for the evaluation of the theoretical intensity function and the determination of the fitting parameters using gnuplot's existing Marquardt-Levenberg nonlinear least squares algorithm [12, 13]. The same program is also used for plotting the data and theoretical functions.

The most important modifications of the program gnuplot are:

- the program is supplemented with new functions to compute the value of the σ-integrals by using the formulae of Budding [1] or Dorren [2], so no (slow) user defined functions are needed for this purpose. The functions are optimized for CPU time. Here we note that the CPU time requirement of the evaluation of the function σ_c using Budding's or Dorren's formulae is the same (within 10 % accuracy).

- the program replots the data and theoretical functions after each step of the iteration, so one can trace continuously how the theoretical function approaches the observed data.

In addition of gnuplot, the program package consists of numerous small scripts. These scripts are mainly used for the file operations. The language of these scripts is *the shell zsh* [8] and use many of the popular GNU utilities [9], for example the program gawk (*GNU awk*).

The program has a graphical interface written in the *Tcl/Tk* scripting language [10]. Using this *frontend*, the user can specify the input/output file names, select the photometric spot model, tune the initial value of the parameters, start the fitting procedure and wiew the results.

The initial or resultant spots are drawn by using the program *povray* [11].

2.2 The functions of the program

2.2.1 File operations

The main panel of the program is shown in Fig. 3. The file names on the left side can be divided into two groups:

- *Input file names*: here can be specified the observed data file containing the intensities (*data file*) and the file containing the initial parameter values (*ini file*). The initial parameter values can be specified on the right side of the panel and can be saved using the button *Save ini file* into the file specified on the field *ini file*. The (previously saved) initial values can be loaded from the file specified on the field *ini file* using the button *Load ini file*.

- *Output file names*:
  - *param. file*: holds the resulting physical parameters. If the standard spot model was used, then both the refined parameters and their asymptotic standard errors are given. If time series mode was used, then the resultant fitting parameters are given in three columns. The first column contains the time value corresponding to the center of the fitting interval, the second gives the parameter value and the third shows the asymptotic standard error of that parameter. The button *View params* shows the parameters in a new window.
  - *info file*: contains statistical information about the fitting (e.g. parameter values, asymptotic standard errors, correlation matrix of the fitting parameters). The button *View info* shows these information in a new window.
  - *curve file (ascii)*: contains the refined theoretical intensity curve (ASCII format), that can be used for plotting the results.
  - *curve file (ps)*: this PostScript file contains the data and the final theoretical intensity curve that can be seen by clicking on the button *View fitted* (see for example Fig.4).
  - *par. curves file*: if time series mode was used, the parameter values in function of time are plotted in this PostScript file. Clicking on the *View p. curves* button a series of PostScript figures with the resulting parameters appear (see for example Fig. 5).
  - *spots image file*: the GIF file representing the spots (image or animation, depending on the spot model). Clicking the *View* button of the spots panel (see later in 2.2.7) shows it.

Two other buttons are used for file operations: the *Cleanup* removes the unnecessary new files and *Exit* terminates the program.

2.2.2 Specification of the initial values of the parameters

The values of the physical parameters and the initial values of the fitting parameters can be specified on the right side of the main panel (Fig. 3). These are the following:

- *USI*: the unspotted intensity of the star in magnitude scale.

- *U*: the U intensity of eq. 5 or 6 which is proportional to the unspotted intensity of the star (in most cases it can be set at unity).

- *L1*: the L_1 intensity of eq. 6 (the intensity of the spotted star), for single stars it is equal to U.
- *kw*: the κ_w coefficient of eq. 5 or 6 is the flux ratio of the spot and the unspotted photosphere at wavelength w.
- *u*: the u linear limb darkening coefficient of eq. 4.
- *Epoch*: the epoch E in eq. 2.
- *Period*: the rotational period of the star.
- *i*: inclination.
- *lambda*: the $\lambda_1, \lambda_2, \lambda_3$ longitudes of the three spots.
- *beta*: the $\beta_1, \beta_2, \beta_3$ latitudes of the three spots.
- *gamma*: the $\gamma_1, \gamma_2, \gamma_3$ angular sizes of the three spots.

Most of these parameters can be added to the list of fitting parameters by pressing the checkbutton located on the right side of the parameter value. In case of models with one or two spot(s), the unnecessary spot(s) can be eliminated by setting the value of γ to 0 and by switching off the fitting of all the spot parameters that belong(s) to the unnecessary spot(s) using the corresponding checkbuttons.

2.2.3 Viewing the initial intensity curve

The time and intensity range used for fitting and plotting the data and theoretical intensity curve can be specified on the right side of the main panel (Fig. 3) by filling out the fields *Time range* and *Intens. range*. By pressing the button *View initial* the program plots the observed data, the theoretical intensity curve corresponding to the actual value of the parameters and the value of USI. In this way, by probing the parameter values, one can setup the approximate value of the parameters which is suitable for the refinement. Fig. 6 shows an example for plotting the initial intensity curve.

2.2.4 The standard spot model

The user has to specify (beside the initial value of the parameters) the convergence criteria of the Marquardt-Levenberg algorithm: the fitting procedure stops if the relative change of χ^2 between two iteration steps is smaller than the limit specified with *Limit for stopping* or if the number of iteration steps reaches the limit specified with *Maximal number of iterations*. The refinement can be started by selecting the option *Use standard spot model* and by clicking on the button *Run FIT*. The program replots the data and theoretical functions after each step of the iteration (see for example Fig. 6), so one can trace continuously how the theoretical function approaches the observed data. After the fit converges, the observed data, the theoretical intensity curve and the value of USI is plotted in function of phase as well. The user can view the output files (see: 2.2.1) or the spots by activating the spots panel with the button *View spots*. Fig. 7 shows an example of the standard spot model with one, two and three spots.

2.2.5 Time series spot model

In addition of the parameters of the standard spot model, two special parameters should be set. The time interval is divided into smaller subintervals of length *Iterated period*. The lower limit of the first subinterval is equal to the lower limit of the entire (time) interval. The limits of the other subintervals are derived from the limits of the previous subinterval shifted by *Step* time. The unit of the parameters *Iterated period* and *Step* is the P period of the star. If the option *Reinitialize parameters* is set, the program initializes the value of the fitting parameters using the initial values (specified on the main panel) in each step of the fitting procedure, so the subsequent fittings are independent. Otherwise the parameter values of the first fitting are initialized using the initial values (specified on the main panel), and the initial parameter values of the other fittings are the resultant parameter values of the previous fitting. The advantage of the latter option is that usually the fitting becomes faster, and if the parameter values change slowly in the function of time, then each fitting starts with good initial values. The refinement can be started by selecting the option *Use time series of the fitting parameters* and by clicking on the button *Run FIT*. Fig. 8 shows two phases of the subinterval fitting. Fig. 4 is an example of the fitted curve and Fig. 5 shows an example of the parameter curves.

2.2.6 Spot temperature determination

If multicolor data are available, then in addition to the spots' coordinates and sizes (see: 1.3) their temperature can also be determined. The program supposes the same temperature for all spots. This case two light curve files should be specified on the field *data files*. The corresponding instrumental wavelengths, USI magnitudes and u linear limb darkening coefficients should be specified on the fields *wavelength*, *USI* and u, respectively. The temperature of the photosphere should be specified on the field *Tstar* and the initial value of the spot temperature should be specified on the field *Tspot*. The unit of these temperatures is 10000 K. The spot temperature can be added to the list of fitting parameters by using the checkbutton on the right side of the field *Tspot*. The refinement can be started by selecting the option *Spot temperature modelling* and by clicking on the button *Run FIT*. The time values of the second light curve and theoretical intensity functions are shifted by the length of the fitting interval specified on the field *Time range*. This way eq. 9 is satisfied. Fig. 9 shows an example of the simultaneous fit of the data made in two bandpasses. After the fit converges, the color index and the difference of the theoretical functions corresponding to the two bands are plotted in function of time and phase as well (see Fig. 10).

2.2.7 Viewing the spots

The spot panel (see Fig. 11 and Fig. 12) can be activated using the *View spots* button of the main panel (Fig. 3).

By selecting the option *View the initial spots*, the program plots the spots corresponding to the initial parameters. The spots can be viewed from an arbitrary direction given by the (*lambda*, *beta*) coordinates (*View from* option) or from the direction of the line of sight at arbitrary time given by *Time* (*View from the line of the sight* option). If the option *Show: axes* is selected, the xyz axes are plotted too and if the option *Show: circles* is selected, the program plots a grid of longitudes and latitudes. The image size can be specified by filling the labels *width* and *height*. The *background color* of the image can be *white* or *black*. The program creates a GIF image file.

- If the standard spot model was used:
  By selecting the option *View the fitted spots*, the program plots the spots corresponding to the resultant parameters of the fit. However, if the option *Show rotation (from the line of sight)* is selected, the program creates a GIF animation showing the rotation of the star with the resulting spot(s). Fig. 13 shows an example for the image created by SML.

- If time series were used:
  This case the program creates a GIF animation showing the resulting spots of the fit viewed from the direction of the line of the sight. The frames of the animation correspond to the mean time of the fitting intervals of the model. The web page [6] contains an example for

the animation: in addition of the rotation of the star, the movement, growth and diminution of the spots can be observed.

The preparation of the image (or animation) can be started using the button *Compute* and the resultant image (or animation) file can be viewed (or played) by pressing the button *View*. The panel can be deactivated using the button *Close*.

3 Downloading and installing SpotModeL

The program works under UNIX/Linux operating systems. Downloading SpotModeL needs about 10 Mbytes disk space. Installing and configuring needs additional disk space between 10 and 100 Mbytes free disk space. The program can be found at the following URL:

`http://www.konkoly.hu/staff/ribarik/SML`

For installing SpotModeL please read the file INSTALL. To start the program simply type:

./sml $< file >$ (data file name with or without extension)

This manual can be downloaded from:

`http://www.konkoly.hu/Mitteilungen/otn12.ps.Z`

References

[1] Budding, E. (1977). *Astroph. and Space Sci.* **48**, 207.

[2] Dorren, J.D. (1987). *Astroph. J.* **320**, 756-767.

[3] Bartus, J. (1996). *Occasional Technical Notes at Konkoly Observatory* No. 1/96

[4] Strassmeier, K.G., Bopp B.W. (1992) *Astron. Astrophys.* **259**, 183

[5] Strassmeier, K.G. and Oláh, K. (1992). *Astronomy and Astrophysics* **259**, 595.

[6] `http://www.konkoly.hu/staff/ribarik/SML`

[7] `http://www.gnuplot.org`

[8] `http://www.zsh.org`

[9] `http://www.gnuplot.info`

[10] `http://www.scriptics.com`

[11] `http://www.povray.org`

[12] Levenberg, K. (1944). *Quart. Appl. Math.* **2**, 164.

[13] Marquardt, D. W. (1963). *SIAM J. Appl. Math.* **11**, 431.

4 Figures

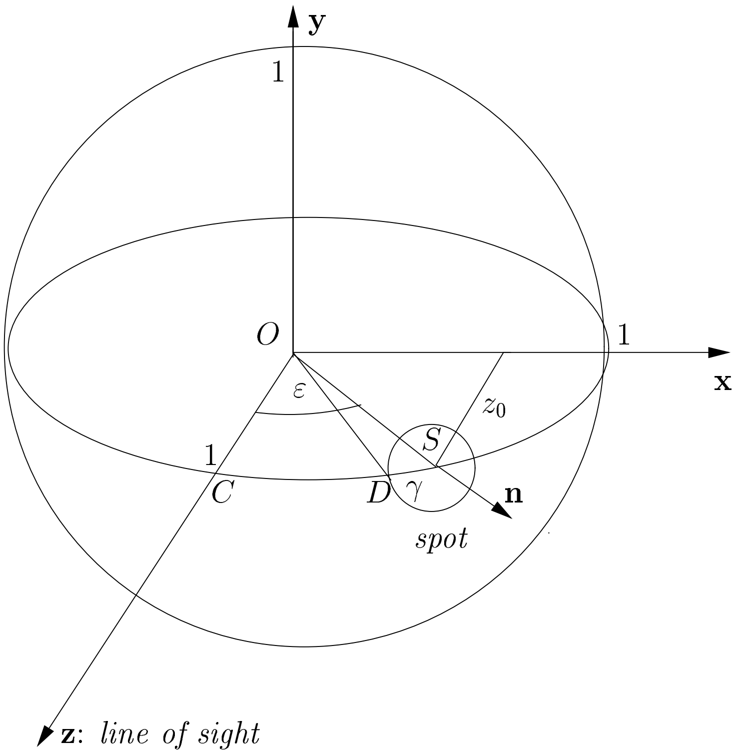

Figure 1: The calculation of the loss of light due to the presence of the spot. $\gamma = SOD\measuredangle$ is the radius of the spot, $\varepsilon = SOC\measuredangle$ is the angle between the surface vector **n** at spot center S and the direction **z** of the line of sight.

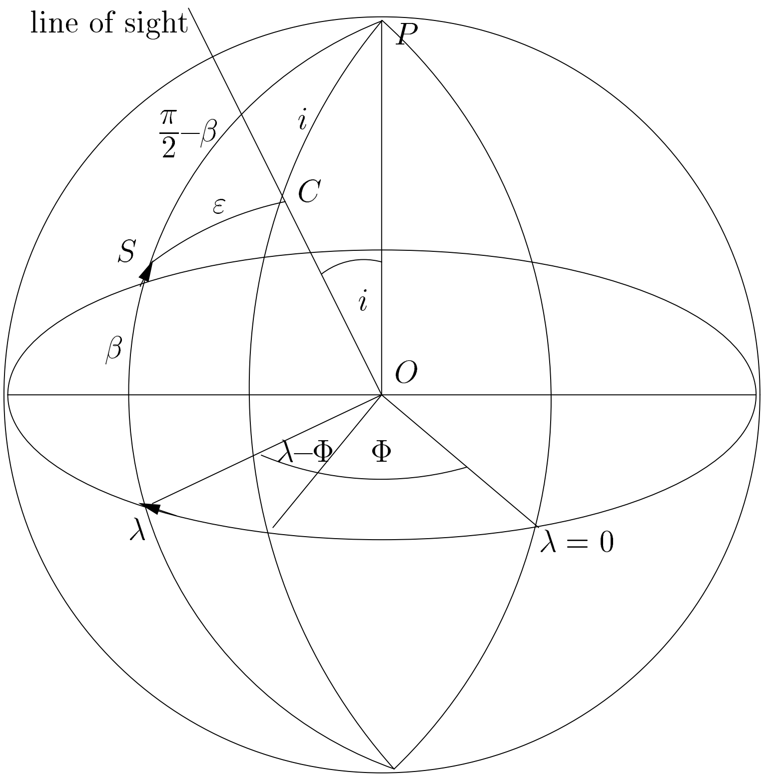

Figure 2: The calculation of the angle ε between the surface vector at spot center S and the direction of the line of sight.

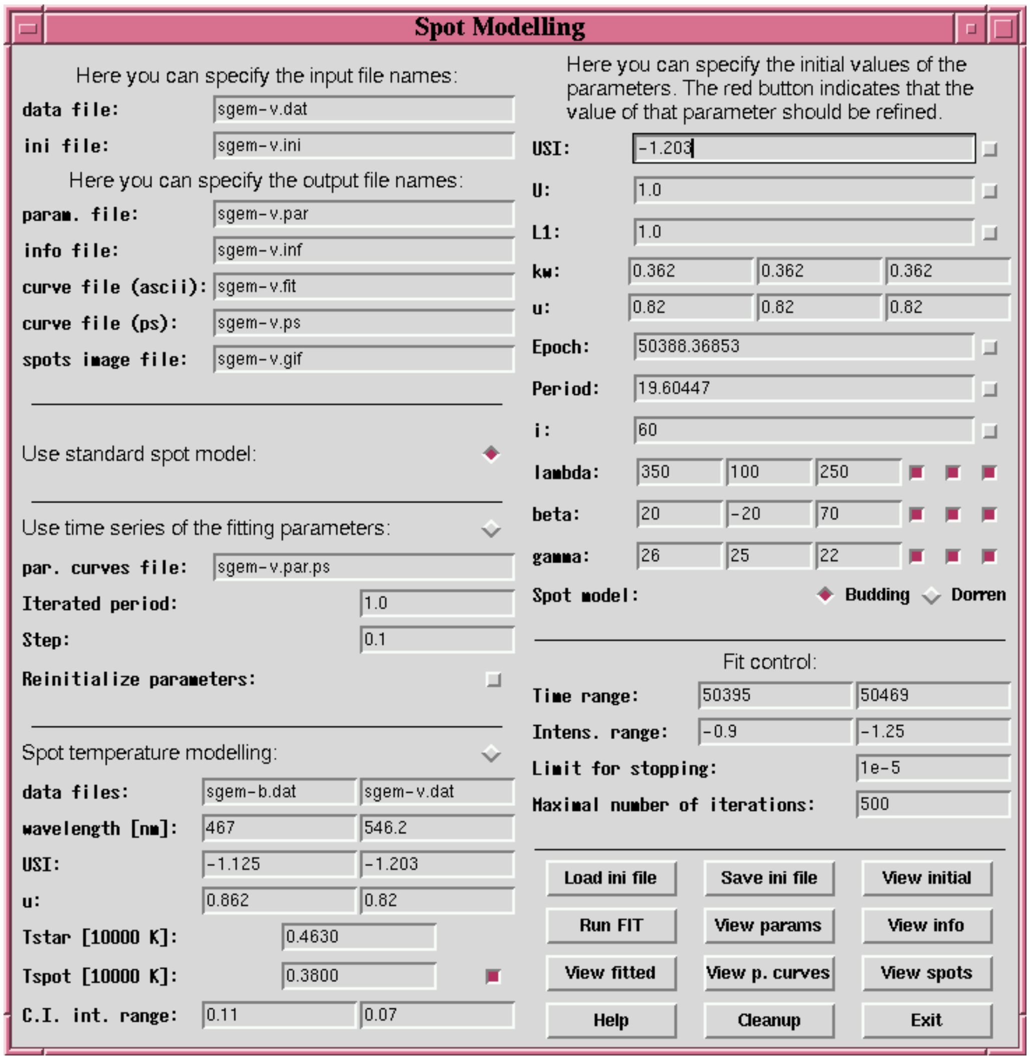

Figure 3: The main panel of the program.

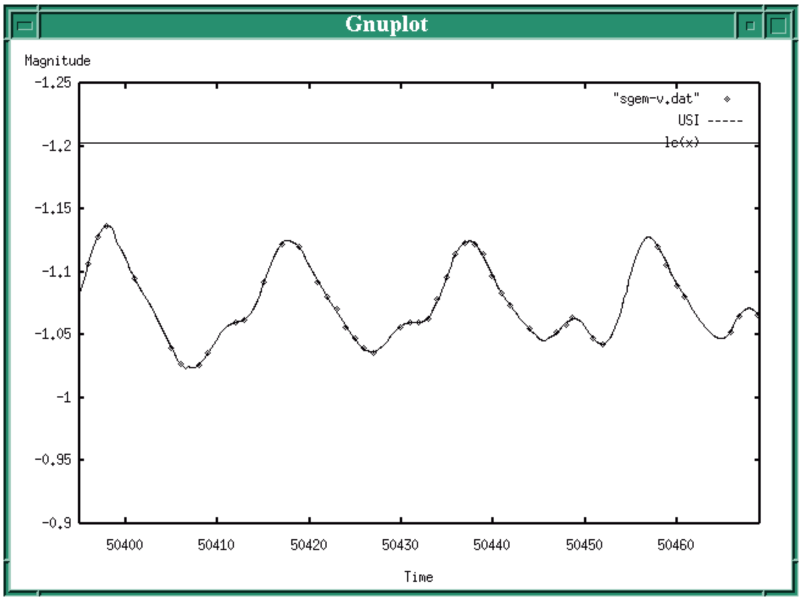

Figure 4: The observed and fitted light curve of σ Gem in band y using time series with three spots.

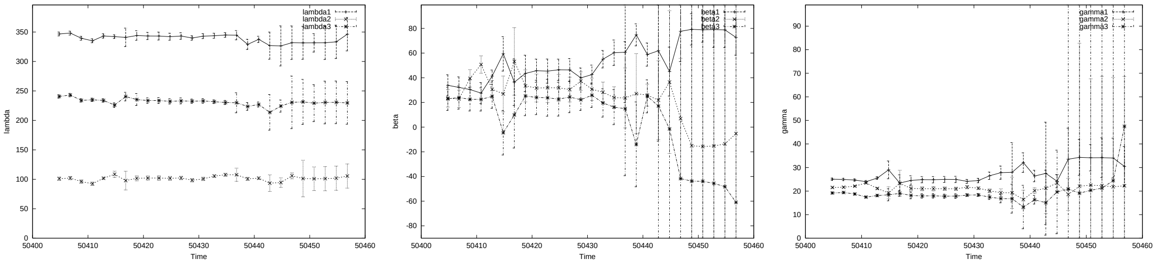

Figure 5: The resultant spot parameters of the time series analysis of the light curve of σ Gem in band y. In this model three spots were used. The errors of the parameters are represented using "bar" symbols.

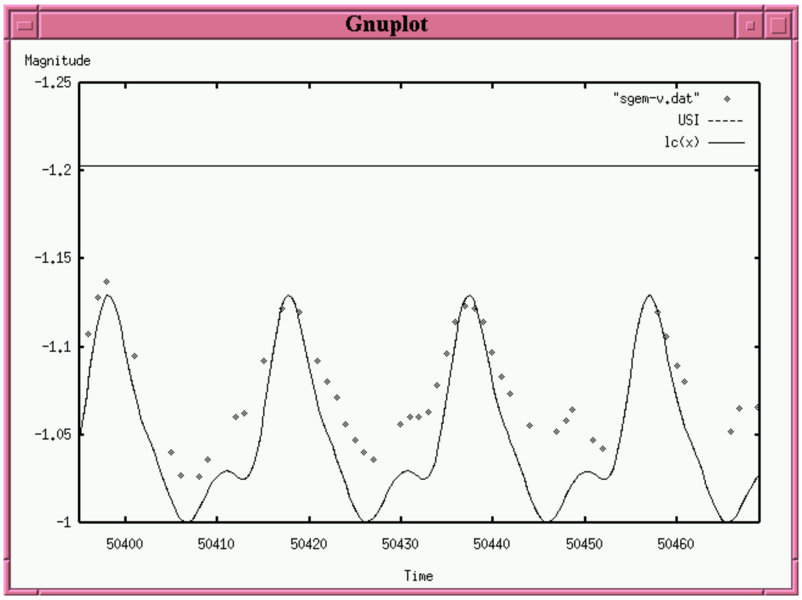

Figure 6: An example for plotting the initial intensity curve.

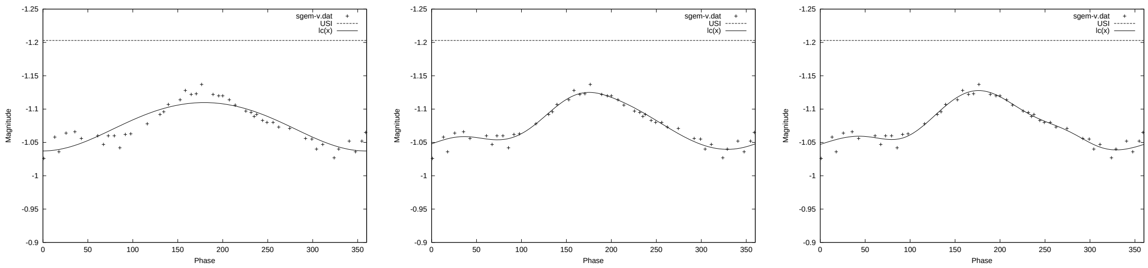

Figure 7: The observed and fitted light curve of σ Gem in band y using the standard spot model with one, two and three spots, respectively.

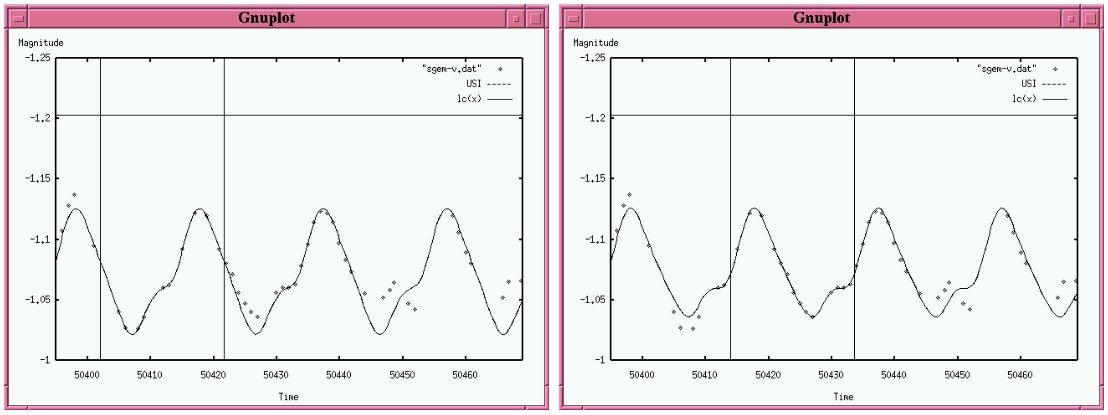

Figure 8: Example for the time series fit. The program takes into account parts of the data of length *Iterated period* in the fitting, shown between two vertical lines. The fitted interval goes through the whole observed time interval with steps *Step*. The figure shows two phases of the fitting.

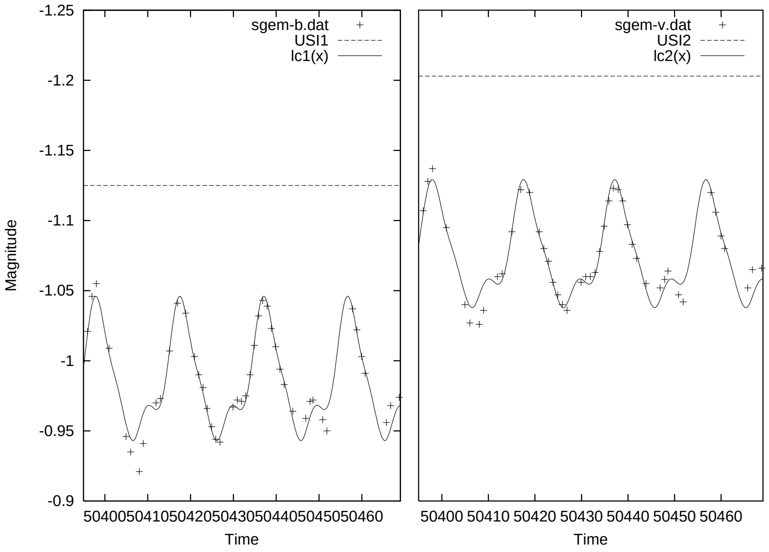

Figure 9: The b and y observations of σ Gem fitted with a three-spot model together with temperature determination.

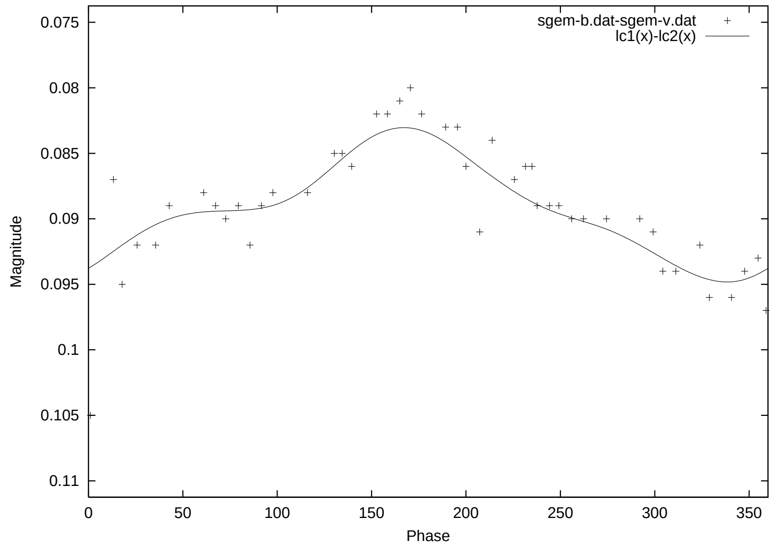

Figure 10: Phase plot of the difference of the b and y observations of σ Gem, i.e. the colour index data, and the fit with a three-spot model together with temperature determination.

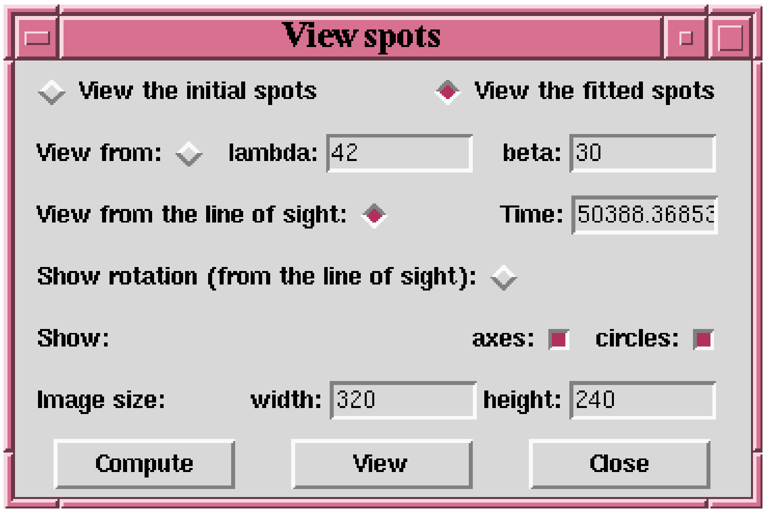

Figure 11: The spots panel if the standard model was used.

Figure 12: The spots panel if time series were used.

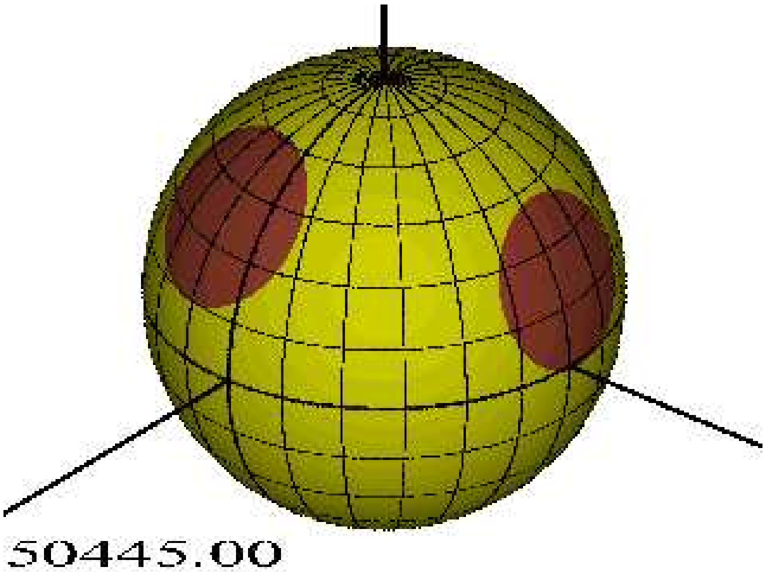

Figure 13: Example for drawing the resulting spots as viewed from the line of sight. The spots resulted from modelling σ-Gem y data with a standard three-spot fit. the vertical axis points to the pole, the longitude of the thick latitude circle is $\lambda = 0\,$deg.